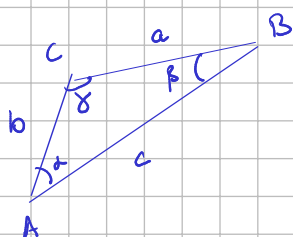


Triangles quelconques - solutions détaillées

Exercice 15



Thm du cosinus: $a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$$

Thm du sinus:

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

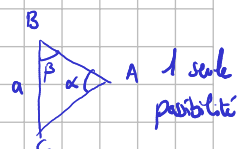
• $a = 7,5$ $\alpha = 45,76^\circ$ $\beta = 110,58^\circ$

$$\gamma = 180^\circ - (45,76^\circ + 110,58^\circ) = \underline{23,66^\circ}$$

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \Rightarrow b = \frac{a \cdot \sin(\beta)}{\sin(\alpha)} = \frac{7,5 \cdot \sin(110,58^\circ)}{\sin(45,76^\circ)} = \underline{9,8}$$

$$c = \frac{a \cdot \sin(\gamma)}{\sin(\alpha)} = \frac{7,5 \cdot \sin(23,66^\circ)}{\sin(45,76^\circ)} = \underline{4,2}$$

le thm du sinus
peut être utilisé



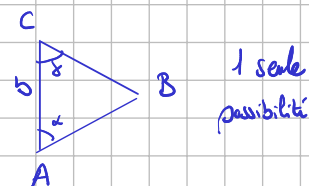
• $b = 8,22$ $\alpha = 39,07^\circ$ $\gamma = 83,96^\circ$

$$\beta = 180^\circ - (39,07^\circ + 83,96^\circ) = \underline{56,97^\circ}$$

$$a = \frac{b \cdot \sin(\alpha)}{\sin(\beta)} = \frac{8,22 \cdot \sin(39,07^\circ)}{\sin(56,97^\circ)} = \underline{6,18}$$

$$c = \frac{b \cdot \sin(\gamma)}{\sin(\beta)} = \frac{8,22 \cdot \sin(83,96^\circ)}{\sin(56,97^\circ)} = \underline{9,15}$$

le thm du sinus
peut être utilisé



• $a = 15,83$ $b = 12,41$ $\alpha = 75,59^\circ$

$$\frac{b}{\sin(\beta)} = \frac{a}{\sin(\alpha)} \Rightarrow \sin(\beta) = \frac{b \cdot \sin(\alpha)}{a} \Rightarrow$$

$$\beta = \sin^{-1}\left(\frac{b \cdot \sin(\alpha)}{a}\right) = \sin^{-1}\left(\frac{12,41 \cdot \sin(75,59^\circ)}{15,83}\right) = \underline{49,4^\circ}$$

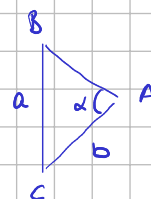
$$\beta_1 = 49,4^\circ$$

$$\gamma_1 = 180^\circ - (75,59^\circ + 49,4^\circ) = \underline{55,01^\circ}$$

$$c = \frac{a \cdot \sin(\gamma)}{\sin(\alpha)} = \frac{15,83 \cdot \sin(55,01^\circ)}{\sin(75,59^\circ)}$$

$$= \underline{13,39}$$

le thm du sinus
peut être utilisé



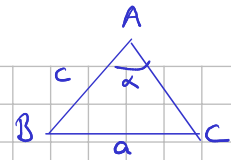
$$\beta_2 = 180^\circ - 49,4^\circ = 130,6^\circ$$

$$\gamma_2 = 180^\circ - (75,59^\circ + 130,6^\circ) = -26,19^\circ$$

pas possible

• $a = 13,6$ $c = 24,35$ $\alpha = 30,28^\circ$

le thm du sinus
peut être utilisé



$$\frac{a}{\sin(\alpha)} = \frac{c}{\sin(\gamma)} \Rightarrow \sin(\gamma) = \frac{c \cdot \sin(\alpha)}{a}$$

$$\Rightarrow \gamma = \sin^{-1}\left(\frac{24,35 \cdot \sin(30,28^\circ)}{13,6}\right) = 64,53^\circ$$

$$\gamma_1 = \underline{\underline{64,53^\circ}}$$

$$\gamma_2 = 180^\circ - 64,53^\circ = \underline{\underline{115,47^\circ}}$$

$$\beta_1 = 180^\circ - (30,28^\circ + 64,53^\circ) = \underline{\underline{85,19^\circ}}$$

$$\beta_2 = 180^\circ - (30,28^\circ + 115,47^\circ) = \underline{\underline{34,25^\circ}}$$

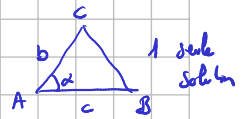
$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \Rightarrow b = \frac{a \cdot \sin(\beta)}{\sin(\alpha)}$$

$$b_1 = \frac{13,6 \cdot \sin(85,19^\circ)}{\sin(30,28^\circ)} = \underline{\underline{26,88}}$$

$$b_2 = \frac{13,6 \cdot \sin(34,25^\circ)}{\sin(30,28^\circ)} = \underline{\underline{15,18^\circ}}$$

• $b = 11,24$ $c = 6,03$ $\alpha = 15,62^\circ$

le thm du sinus ne peut
pas être utilisé!



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha) = 11,24^2 + 6,03^2 - 2 \cdot 11,24 \cdot 6,03 \cdot \cos(15,62^\circ) = 32,15$$

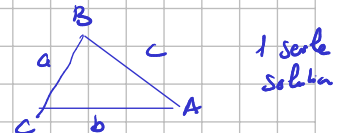
$$a = \sqrt{32,15} = \underline{\underline{5,67}}$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta) \Rightarrow \cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$\beta = \cos^{-1}\left(\frac{5,67^2 + 6,03^2 - 11,24^2}{2 \cdot 5,67 \cdot 6,03}\right) = \underline{\underline{147,74^\circ}}$$

$$\gamma = 180^\circ - (147,74^\circ + 15,62^\circ) = \underline{\underline{16,64^\circ}}$$

• $a = 13$ $b = 14$ $c = 15$ utiliser le thm du cosinus



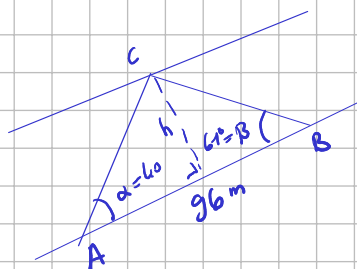
$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha) \Rightarrow \cos(\alpha) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\alpha = \cos^{-1}\left(\frac{14^2 + 15^2 - 13^2}{2 \cdot 14 \cdot 15}\right) = \underline{\underline{53,13^\circ}}$$

$$\beta = \cos^{-1}\left(\frac{13^2 + 15^2 - 14^2}{2 \cdot 13 \cdot 15}\right) = \underline{\underline{59,49^\circ}}$$

$$\gamma = 180^\circ - (53,13^\circ + 59,49^\circ) = \underline{\underline{67,38^\circ}}$$

Exercice 16



$$\gamma = 180^\circ - (40^\circ + 61^\circ) = 180^\circ - 101^\circ = 79^\circ$$

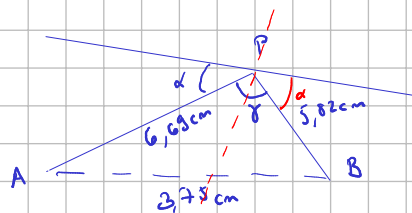
Théorème du sinus:

$$\frac{96}{\sin(79^\circ)} = \frac{AC}{\sin(61^\circ)} = \frac{CB}{\sin(40^\circ)}$$

$$\Rightarrow AC = \frac{96 \cdot \sin(61^\circ)}{\sin(79^\circ)} = 85,5 \text{ m}$$

Trigo ds le Δ rectangle: $\sin(\alpha) = \frac{h}{AC} \Rightarrow h = AC \cdot \sin(\alpha) = 85,5 \cdot \sin(40^\circ) = \underline{\underline{54,98 \text{ m}}}$

Exercice 17



Théorème du cosinus: $AB^2 = AP^2 + PB^2 - 2 \cdot AP \cdot PB \cdot \cos(\gamma)$

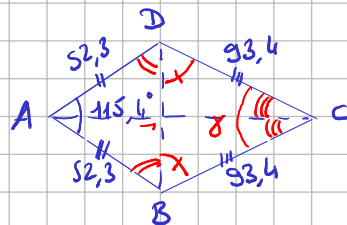
$$\cos(\gamma) = \frac{AP^2 + PB^2 - AB^2}{2 \cdot AP \cdot PB}$$

$$\cos(\gamma) = \frac{6,69^2 + 5,82^2 - 3,75^2}{2 \cdot 6,69 \cdot 5,82} = 0,8291$$

$$\gamma = \cos^{-1}(0,8291) = 33,99^\circ$$

$$\alpha = \frac{180^\circ - 33,99^\circ}{2} = \underline{\underline{73^\circ}}$$

Exercice 18



a) $\widehat{DBA} = \widehat{ADB} = 180^\circ - (90^\circ + \frac{115,4^\circ}{2}) = 32,3^\circ$

Thm du cosinus:

$$DB^2 = 52,3^2 + 52,3^2 - 2 \cdot 52,3 \cdot 52,3 \cdot \cos(115,4^\circ) = 7817,1$$

$$DB = \sqrt{7817,1} = 88,41$$

$$\cos(\gamma) = \frac{93,4^2 + 93,4^2 - 88,41^2}{2 \cdot 93,4 \cdot 93,4} = 0,55 \Rightarrow \gamma = \cos^{-1}(0,55) = \underline{\underline{56,5^\circ}}$$

$$\widehat{CBD} = \widehat{BDC} = 180^\circ - (90^\circ + \frac{56,5^\circ}{2}) = 61,75^\circ$$

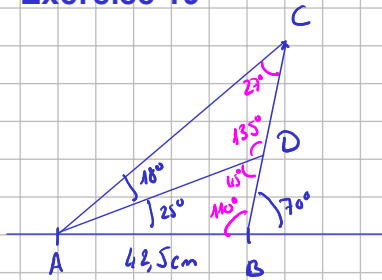
$$\widehat{CBA} = \widehat{ADC} = 32,3^\circ + 61,75^\circ = \underline{\underline{94,05^\circ}}$$

b) $AC^2 = 52,3^2 + 93,4^2 - 2 \cdot 52,3 \cdot 93,4 \cdot \cos(94,05^\circ) = 12149,2123$

$$AC = \sqrt{12149,2} = 110,22$$

$$\text{Aire: } \frac{DB \cdot AC}{2} = \frac{88,41 \cdot 110,22}{2} = \underline{\underline{4872,7}}$$

Exercice 19



$$\beta = 180^\circ - 70^\circ = 110^\circ, \quad \delta = 180^\circ - 110^\circ - 25^\circ = 45^\circ$$

$$\delta' = 180^\circ - 45^\circ = 135^\circ, \quad \gamma = 180^\circ - 18^\circ - 25^\circ - 110^\circ = 27^\circ$$

Thm du sinus ds ABD :

$$\frac{42,5}{\sin(45^\circ)} = \frac{DB}{\sin(25^\circ)} = \frac{AD}{\sin(110^\circ)}$$

$$\Rightarrow DB = \frac{42,5 \cdot \sin(25^\circ)}{\sin(45^\circ)} = \underline{\underline{25,4 \text{ cm}}}$$

$$AD = \frac{42,5 \cdot \sin(110^\circ)}{\sin(45^\circ)} = 56,5 \text{ cm}$$

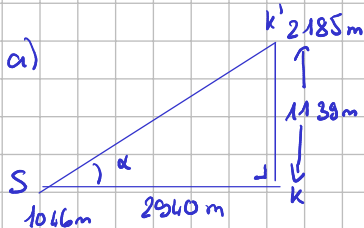
Thm du sinus ds ABC :

$$\frac{42,5}{\sin(27^\circ)} = \frac{BC}{\sin(18+25^\circ)} = \frac{AC}{\sin(110^\circ)}$$

$$\Rightarrow BC = \frac{42,5 \cdot \sin(43^\circ)}{\sin(27^\circ)} = \underline{\underline{63,8 \text{ cm}}}$$

$$AC = \frac{42,5 \cdot \sin(110^\circ)}{\sin(27^\circ)} = \underline{\underline{88 \text{ cm}}}$$

Exercice 20

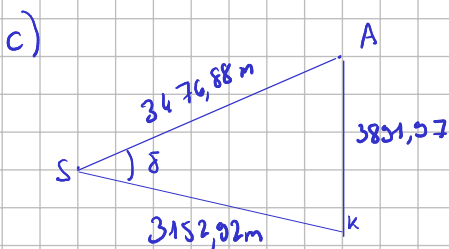


	SK	SA	AK
cote [cm]	4,2	4,9	5,5
riel [cm]	70'000	294'000	343'000
	2,94 km	3,43 km	3,85 km

$$\sin(\alpha) = \frac{1139}{2940} \Rightarrow \alpha = \tan^{-1}\left(\frac{1139}{2940}\right) = \underline{\underline{21,18^\circ}}$$

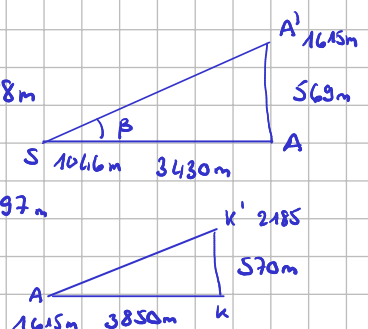
b)

$$\sin(\alpha) = \frac{KK'}{SK'} \Rightarrow SK' = \frac{KK'}{\sin(\alpha)} = \frac{1139}{\sin(21,18^\circ)} = \underline{\underline{3152,92 \text{ m}}}$$



$$SA' = \frac{569}{\sin\left(\tan^{-1}\left(\frac{569}{3430}\right)\right)} = 3476,88 \text{ m}$$

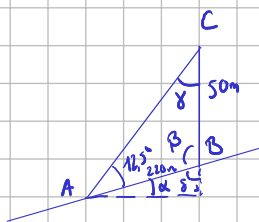
$$AK' = \frac{570}{\sin\left(\tan^{-1}\left(\frac{570}{3850}\right)\right)} = 3891,97 \text{ m}$$



$$AK^2 = SA^2 + SK^2 - 2 \cdot SA \cdot SK \cdot \cos(\delta)$$

$$\cos(\delta) = \frac{SA^2 + SK^2 - AK^2}{2 \cdot SA \cdot SK} = 0,31 \Rightarrow \delta = \underline{\underline{71,71^\circ}}$$

Exercise 21



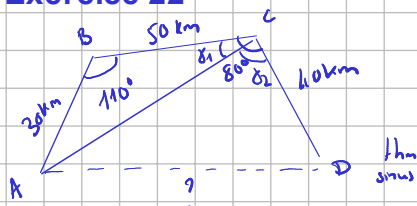
$$\frac{50}{\sin(12.5^\circ)} = \frac{220}{\sin(\gamma)} \Rightarrow \gamma = \sin^{-1}\left(\frac{220 \cdot \sin(12.5^\circ)}{50}\right) = 72.24^\circ$$

$$\beta = 180^\circ - 12.5^\circ - 72.24^\circ = 95.26^\circ$$

$$\delta = 180^\circ - 95.26^\circ = 84.74^\circ$$

$$\alpha = 180^\circ - 90^\circ - 84.74^\circ = \underline{\underline{5.3^\circ}}$$

Exercise 22



$$AC^2 = 30^2 + 50^2 - 2 \cdot 30 \cdot 50 \cdot \cos(110^\circ) \quad \text{Thm du cosinus}$$

$$AC = \sqrt{4426.06} = 66.53 \text{ km}$$

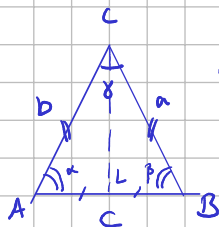
$$\frac{66.53}{\sin(110^\circ)} = \frac{30}{\sin(\gamma_1)} \Rightarrow \gamma_1 = \sin^{-1}\left(\frac{30 \cdot \sin(110^\circ)}{66.53}\right) = 25.07^\circ$$

$$\gamma_2 = 80^\circ - 25.07^\circ = 54.93^\circ$$

$$\text{Thm du cosinus: } AD^2 = 66.53^2 + 40^2 - 2 \cdot 66.53 \cdot 40 \cdot \cos(54.93^\circ)$$

$$AD = \sqrt{2967.93} = \underline{\underline{54.48 \text{ km}}}$$

Exercise 23



$$a) \quad \cos(\beta) = \frac{c}{2c} \Rightarrow \cos(\beta) = \frac{1}{4} \Rightarrow \alpha = \beta = \cos^{-1}\left(\frac{1}{4}\right) \approx \underline{\underline{75.5^\circ}}$$

$$\gamma = 180^\circ - 2 \cdot 75.5^\circ = \underline{\underline{29^\circ}}$$

$$b) \quad c^2 = a^2 + a^2 - 2a \cdot a \cdot \cos(\gamma) \Rightarrow c^2 = 2a^2 - 2a^2 \cos(\gamma) = 2a^2(1 - \cos(\gamma))$$

$$c) \quad a^2 = a^2 + c^2 - 2ac \cdot \cos(\alpha) \Rightarrow 0 = c^2 - 2ac \cos(\alpha)$$

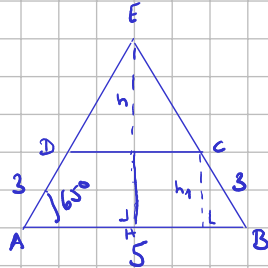
$$d) \quad 0 = c^2 - 2ac \cdot \cos(\alpha) \Rightarrow 2ac \cdot \cos(\alpha) = c^2 \Rightarrow a = \frac{c^2}{2c \cdot \cos(\alpha)} = \frac{c}{2 \cdot \cos(\alpha)} = \frac{6}{2 \cdot \cos(54.9^\circ)} \approx 5.1$$

$$e) \quad 0 = c^2 - 2 \cdot 10.5 \cdot c \cdot \cos(38^\circ) \Leftrightarrow 0 = c(c - 21 \cdot \cos(38^\circ))$$

$$\Leftrightarrow c = 0 \quad \text{à exclure}$$

$$\underline{\underline{c = 21 \cdot \cos(38^\circ) \approx 16.54}}$$

Exercise 24

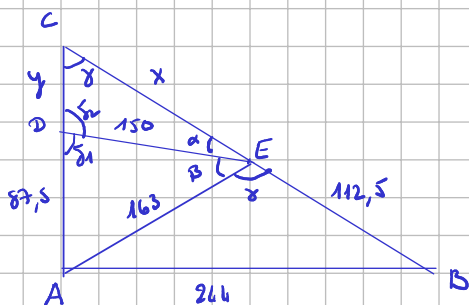


$$\tan(65^\circ) = \frac{EH}{\frac{5}{2}} \Rightarrow EH = \frac{5}{2} \cdot \tan(65^\circ) = 5,36$$

$$\sin(65^\circ) = \frac{h_1}{3} \Rightarrow h_1 = 3 \cdot \sin(65^\circ) = 2,72$$

$$h = 5,36 - 2,72 = \underline{\underline{2,64}}$$

Exercise 25



$$\text{Thm du cosinus: } 244^2 = 163^2 + 112,5^2 - 2 \cdot 163 \cdot 112,5 \cdot \cos(\alpha)$$

$$\cos(\alpha) = \frac{163^2 + 112,5^2 - 244^2}{2 \cdot 163 \cdot 112,5} = -0,55 \Rightarrow \alpha = \cos^{-1}(-0,55) = 123,7^\circ$$

$$\cos(\beta) = \frac{150^2 + 163^2 - 87,5^2}{2 \cdot 150 \cdot 163} = 0,85 \Rightarrow \beta = \cos^{-1}(0,85) = 32,13^\circ$$

$$\alpha = 180^\circ - 123,7^\circ - 32,13^\circ = 24,18^\circ$$

$$\cos(\delta_1) = \frac{150^2 + 87,5^2 - 163^2}{2 \cdot 150 \cdot 87,5} = 0,14 \Rightarrow \delta_1 = \cos^{-1}(0,14) = 82,15^\circ$$

$$\delta_2 = 180^\circ - 82,15^\circ = 97,85^\circ$$

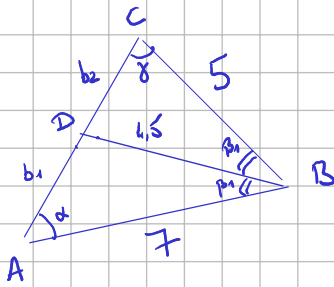
$$\gamma = 180^\circ - 97,85^\circ - 24,18^\circ = 57,97^\circ$$

$$\text{Thm du sinus: } \frac{150}{\sin(57,97^\circ)} = \frac{x}{\sin(97,85^\circ)} = \frac{y}{\sin(24,18^\circ)}$$

$$\Rightarrow x = \frac{150 \cdot \sin(97,85^\circ)}{\sin(57,97^\circ)} \approx \underline{\underline{175,28}}$$

$$\Rightarrow y = \frac{150 \cdot \sin(24,18^\circ)}{\sin(57,97^\circ)} \approx \underline{\underline{72,47}}$$

Challenge



Thm de la bissectrice $\frac{b_1}{b_2} = \frac{7}{5} \Rightarrow b_1 = \frac{7}{5} b_2$

$$b_1 + b_2 = b \Rightarrow \frac{7}{5} b_2 + b_2 = b \Rightarrow \frac{12}{5} b_2 = b \Rightarrow b_2 = \frac{5}{12} b$$

$$\text{et } b_1 = b - b_2 = b - \frac{5}{12} b = \frac{7}{12} b$$

$$b_1^2 = 7^2 + 4,5^2 - 2 \cdot 7 \cdot 4,5 \cdot \cos(\beta_1) \Rightarrow \left(\frac{7}{12} b\right)^2 = 69,25 - 63 \cos(\beta_1)$$

$$b_2^2 = 5^2 + 4,5^2 - 2 \cdot 5 \cdot 4,5 \cdot \cos(\beta_1) \Rightarrow \left(\frac{5}{12} b\right)^2 = 45,25 - 45 \cos(\beta_1)$$

$$\frac{49}{144} b^2 + 63 \cos(\beta_1) = 69,25$$

· 5

$$\frac{25}{144} b^2 + 45 \cos(\beta_1) = 45,25$$

· (-7)

$$\frac{245}{144} b^2 + 315 \cos(\beta_1) = 346,25$$

$$+ \frac{-175}{144} b^2 - 315 \cos(\beta_1) = -316,75$$

$$\frac{70}{144} b^2 = 29,5$$

$$\Rightarrow b^2 = 60,69 \Rightarrow b = \sqrt{60,69} \approx \underline{\underline{7,8}}$$

$$b^2 = 5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cdot \cos(\beta) \Rightarrow \cos(\beta) = \frac{5^2 + 7^2 - 7,8^2}{2 \cdot 5 \cdot 7} \approx 0,19$$

$$\Rightarrow \beta = \cos^{-1}(0,19) \approx \underline{\underline{79^\circ}}$$

$$5^2 = 7^2 + b^2 - 2 \cdot 7 \cdot b \cdot \cos(\alpha) \Rightarrow \cos(\alpha) = \frac{7^2 + b^2 - 5^2}{2 \cdot 7 \cdot b} \approx 0,78$$

$$\Rightarrow \alpha = \cos^{-1}(0,78) \approx \underline{\underline{39,1^\circ}}$$

$$\gamma = 180^\circ - 79^\circ - 39,1^\circ \approx \underline{\underline{61,9^\circ}}$$