Équations trigonométriques - solutions détaillées

Exercice 8

a) $\sin(\varphi) = 0{,}8473 \Rightarrow \varphi_1 = \arcsin(0{,}8473) + k\cdot 360° = 57{,}92° + k\cdot 360°$

$\varphi_2 = 180° - \varphi_1 = 122{,}08° + k\cdot 360° \qquad k\in\mathbb{Z}$

$S = \{57{,}92° + k\cdot 360° \mid k\in\mathbb{Z}\} \cup \{122{,}08° + k\cdot 360° \mid k\in\mathbb{Z}\}$

b) $\cos(\varphi) = -0{,}38 \Rightarrow \varphi_1 = \arccos(-0{,}38) + k\cdot 360° = 112{,}33° + k\cdot 360°$

$\varphi_2 = -\varphi_1 = -112{,}33° + k\cdot 360° \quad , k\in\mathbb{Z}$

$S = \{112{,}33° + k\cdot 360° \mid k\in\mathbb{Z}\} \cup \{-112{,}33° + k\cdot 360° \mid k\in\mathbb{Z}\}$

c) $\tan(\varphi) = -1{,}6 \Rightarrow \varphi = \arctan(-1{,}6) + k\cdot 180° = -57{,}99° + k\cdot 180° \quad k\in\mathbb{Z}$

$S = \{122° + k\cdot 180° \mid k\in\mathbb{Z}\} \qquad = 122° + k\cdot 180°$

d) $\cos(x) = 1{,}352 \qquad \cos(x) \leq 1 \Rightarrow S = \emptyset$

e) $\tan(x) = -0{,}9042 \Rightarrow x = \arctan(-0{,}9042) + k\cdot\pi = -0{,}7351 + k\cdot\pi$

$\xleftarrow{+\pi}$

$= 2{,}4065 + k\pi \qquad k\in\mathbb{Z}$

$S = \{2{,}4065 + k\pi \mid k\in\mathbb{Z}\}$

f) $\sin(x) = 0{,}44 \Rightarrow x_1 = \arcsin(0{,}44) + k\cdot 2\pi = 0{,}4556 + k\cdot 2\pi$

$x_2 = \pi - x_1 = 2{,}686 + k\cdot 2\pi \qquad k\in\mathbb{Z}$

$S = \{0{,}4556 + k\cdot 2\pi \mid k\in\mathbb{Z}\} \cup \{2{,}686 + k\cdot 2\pi \mid k\in\mathbb{Z}\}$

Exercice 9

a) $\cos(\alpha - 60°) = -\frac{1}{2} \qquad \arccos(..)$

$\alpha - 60° = \arccos(-\frac{1}{2}) + k\cdot 360°$

$\alpha_1 - 60° = 120° + k\cdot 360° \qquad +60 \qquad \alpha_2 - 60° = -120° + k\cdot 360°$

$\alpha_1 = 180° + k\cdot 360° \qquad \alpha_2 = -60° + k\cdot 360°$

$S = \{180° + k\cdot 360° \mid k\in\mathbb{Z}\} \cup \{-60° + k\cdot 360° \mid k\in\mathbb{Z}\}$

b) $\tan(4x) = \sqrt{3}$ $\quad\Big|\ \arctan(..)$

$4x = \arctan(\sqrt{3}) + k\cdot\pi$

$4x = \frac{\pi}{3} + k\cdot\pi$ $\quad\Big|\ :4$

$x = \frac{\pi}{12} + k\cdot\frac{\pi}{4}$

$S = \left\{ \frac{\pi}{12} + k\cdot\frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$

c) $2\cdot\sin\left(x + \frac{\pi}{12}\right) = -\sqrt{2}$ $\quad\Big|\ :2$

$\sin\left(x + \frac{\pi}{12}\right) = -\frac{\sqrt{2}}{2}$ $\quad\Big|\ \arcsin(..)$

$x_1 + \frac{\pi}{12} = -\frac{\pi}{4} + k\cdot 2\pi$ $\quad\Big|\ -\frac{\pi}{12}$ $\qquad x_2 + \frac{\pi}{12} = \frac{5\pi}{4} + k\cdot 2\pi$

$x_1 = -\frac{\pi}{3} + k\cdot 2\pi$ $\qquad x_2 = \frac{7\pi}{6} + k\cdot 2\pi$

$S = \left\{ -\frac{\pi}{3} + k\cdot 2\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{7\pi}{6} + k\cdot 2\pi \mid k \in \mathbb{Z} \right\}$

d) $\sin(3\alpha + 15^\circ) = -0{,}5$ $\quad\Big|\ \arcsin(...)$

$3\alpha + 15^\circ = \arcsin(-0{,}5) + k\cdot 360^\circ$

$3\alpha_1 + 15^\circ = -30^\circ + k\cdot 360^\circ$ $\quad\Big|\ -15^\circ$ $\qquad 3\alpha_2 + 15^\circ = 210^\circ + k\cdot 360^\circ \quad\Big|\ -15^\circ$

$3\alpha_1 = -45^\circ + k\cdot 360^\circ$ $\quad\Big|\ :3$ $\qquad 3\alpha_2 = 195^\circ + k\cdot 360^\circ \quad\Big|\ :3$

$\alpha_1 = \frac{-45^\circ + k\cdot 360^\circ}{3}$ $\qquad \alpha_2 = 65^\circ + k\cdot 120^\circ$

$\alpha_1 = -15^\circ + k\cdot 120^\circ$

$S = \{ -15^\circ + k\cdot 120^\circ \mid k \in \mathbb{Z} \} \cup \{ 65^\circ + k\cdot 120^\circ \mid k \in \mathbb{Z} \}$

Exercice 10

a)

$$\sin(3x) = \sin\left(\frac{\pi}{2} - x\right) \quad \Big| \arcsin(..)$$

$$3x_1 = \frac{\pi}{2} - x_1 + k\cdot 2\pi \quad \Big| +x_1$$

$$4x_1 = \frac{\pi}{2} + k\cdot 2\pi \quad \Big| :4$$

$$x_1 = \frac{\pi}{8} + k\cdot\frac{\pi}{2}$$

$$3x_2 = \pi - \left(\frac{\pi}{2} - x_2\right) + k\cdot 2\pi \quad \Big| \text{cl}$$

$$3x_2 = \frac{\pi}{2} + x_2 + k\cdot 2\pi \quad \Big| -x_2$$

$$2x_2 = \frac{\pi}{2} + k\cdot 2\pi \quad \Big| :2$$

$$x_2 = \frac{\pi}{4} + k\cdot\pi$$

$$S = \left\{ \frac{\pi}{8} + k\cdot\frac{\pi}{2} \;\middle|\; k\in\mathbb{Z} \right\} \cup \left\{ \frac{\pi}{4} + k\cdot\pi \;\middle|\; k\in\mathbb{Z} \right\}$$

b)

$$\tan\left(\frac{2\pi}{3} - x\right) = \tan(2x) \quad \Big| \arctan(..)$$

$$\frac{2\pi}{3} - x = 2x + k\pi \quad \Big| -\frac{2\pi}{3} - 2x$$

$$-3x = -\frac{2\pi}{3} + k\pi \quad \Big| :(-3)$$

$$x = \frac{2\pi}{9} + k\cdot\frac{\pi}{3}$$

$$S = \left\{ \frac{2\pi}{9} + k\cdot\frac{\pi}{3} \;\middle|\; k\in\mathbb{Z} \right\}$$

c)

$-\cos(x) = \cos(\pi - x)$

$$\cos\left(\frac{2\alpha}{3} - 45°\right) = -\cos\left(\frac{3\alpha}{2} - 30°\right)$$

$$\cos\left(\frac{2\alpha}{3} - 45°\right) = \cos\left(180° - \left(\frac{3\alpha}{2} - 30°\right)\right)$$

$$\cos\left(\frac{2\alpha}{3} - 45°\right) = \cos\left(210° - \frac{3\alpha}{2}\right)$$

$$\frac{2\alpha}{3} - 45 = 210° - \frac{3\alpha}{2} + k\cdot 360°$$

$$\frac{2\alpha}{3} + \frac{3\alpha}{2} = 255° + k\cdot 360°$$

$$\frac{13\alpha}{6} = 255° + k\cdot 360°$$

$$\alpha_1 = 117{,}69° + k\cdot 166{,}15°$$

$$\frac{2\alpha}{3} - 45 = -\left(210° - \frac{3\alpha}{2}\right) + k\cdot 360°$$

$$\frac{2\alpha}{3} - \frac{3\alpha}{2} = -165° + k\cdot 360°$$

$$-\frac{5\alpha}{6} = -165° + k\cdot 360°$$

$$\alpha_2 = 198° + k\cdot 432°$$

$$S = \left\{ 117{,}69° + k\cdot 166{,}15° \;\middle|\; k\in\mathbb{Z} \right\} \cup \left\{ \underset{-234}{198°} + k\cdot 432° \;\middle|\; k\in\mathbb{Z} \right\}$$

d)

$-\sin(\alpha) = \cos\left(\frac{\pi}{2} + \alpha\right)$

$$\sin\left(\frac{5\alpha}{3}\right) + \cos\left(\frac{\alpha}{2}\right) = 0$$

$$\cos\left(\frac{\alpha}{2}\right) = -\sin\left(\frac{5\alpha}{3}\right)$$

$$\cos\left(\frac{\alpha}{2}\right) = \cos\left(90° + \frac{5\alpha}{3}\right)$$

$$\frac{\alpha}{2} = 90° + \frac{5\alpha}{3} + k\cdot 360°$$

$$-\frac{7}{6}\alpha = 90° + k\cdot 360°$$

$$\alpha_1 = -77{,}14° + k\cdot 308{,}57$$

$$\frac{\alpha}{2} = -\left(90° + \frac{5\alpha}{3}\right) + k\cdot 360°$$

$$\frac{13\alpha}{6} = -90° + k\cdot 360°$$

$$\alpha_2 = -41{,}54° + k\cdot 166{,}15°$$

$$S = \left\{ -77{,}14° + k\cdot 308{,}57 \;\middle|\; k\in\mathbb{Z} \right\} \cup \left\{ \underbrace{-41{,}54°}_{124{,}62°} + k\cdot 166{,}15° \;\middle|\; k\in\mathbb{Z} \right\}$$

Exercice 11

a) $2\sin\left(2\left(x-\frac{\pi}{2}\right)\right)=0 \quad | :2$

$\sin\left(2\left(x-\frac{\pi}{2}\right)\right)=0 \quad | \arcsin(..)$

$2\left(x-\frac{\pi}{2}\right)=0+k\cdot\pi$

$2x-\pi=0+k\cdot\pi$

$2x=\pi+k\cdot\pi \quad | :2$

$x=\frac{\pi}{2}+k\cdot\frac{\pi}{2}=0+k\cdot\frac{\pi}{2}$

$2\left(x-\frac{\pi}{2}\right)=\pi+k\cdot\pi$

$2x-\pi=\pi+k\pi$

$2x=2\pi+k\pi$

$x=\pi+k\cdot\frac{\pi}{2}$

$S=\left\{k\cdot\frac{\pi}{2} \mid k\in\mathbb{Z}\right\}$

b) $3\cdot\cos\left(\frac{x+\pi}{2}\right)=0 \quad | :3$

$\cos\left(\frac{x+\pi}{2}\right)=0 \quad | \arccos(..)$

$\frac{x+\pi}{2}=\frac{\pi}{2}+k\cdot2\pi \quad | \cdot 2$

$x+\pi=\pi+k\cdot4\pi \quad | -\pi$

$x_1=k\cdot4\pi$

$\frac{x+\pi}{2}=-\frac{\pi}{2}+k\cdot2\pi \quad | \cdot 2$

$x+\pi=-\pi+k\cdot4\pi \quad | -\pi$

$x_2=-2\pi+k\cdot4\pi$

$S=\{k\cdot4\pi \mid k\in\mathbb{Z}\}\cup\{-2\pi+k\cdot4\pi \mid k\in\mathbb{Z}\}=\{k\cdot2\pi \mid k\in\mathbb{Z}\}$

Exercice 12

a) $2\sin^2(x)-5\sin(x)+2=0$ Posons $\sin(x)=y$

$2y^2-5y+2=0$

$\Delta=(-5)^2-4\cdot2\cdot2=25-16=9$

$y_{1,2}=\frac{5\pm\sqrt{9}}{2\cdot2}=\frac{5\pm3}{4}$

$y_1=\frac{8}{4}=2 \rightarrow \sin(x)=2$ pas possible

$y_2=\frac{2}{4}=\frac{1}{2} \rightarrow \sin(x)=\frac{1}{2}$

$x_1=\arcsin\left(\frac{1}{2}\right)+k\cdot2\pi=\frac{\pi}{6}+k\cdot2\pi$

$x_2=\pi-\frac{\pi}{6}+k\cdot2\pi=\frac{5\pi}{6}+k\cdot2\pi \qquad k\in\mathbb{Z}$

$S=\left\{\frac{\pi}{6}+k\cdot2\pi \mid k\in\mathbb{Z}\right\}\cup\left\{\frac{5\pi}{6}+k\cdot2\pi \mid k\in\mathbb{Z}\right\}$

b) $\tan^4(x) - 4\tan^2(x) = -3$ posons $y = \tan^2(x)$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y - 3 = 0$ ou $y - 1 = 0$

$\tan^2(x) - 3 = 0$ $\tan^2(x) - 1 = 0$

$(\tan(x)+\sqrt{3})(\tan(x)-\sqrt{3}) = 0$ $(\tan(x)+1)(\tan(x)-1) = 0$

$\tan(x) = -\sqrt{3}$ $\tan(x) = \sqrt{3}$ $\tan(x) = -1$ $\tan(x) = 1$

$x_1 = \arctan(-\sqrt{3}) + k\cdot\pi$ $x_2 = \arctan(\sqrt{3}) + k\cdot\pi$ $x_3 = \arctan(-1) + k\cdot\pi$ $x_4 = \arctan(1) + k\cdot\pi$

$x_1 = -\frac{\pi}{3} + k\pi$ $x_2 = \frac{\pi}{3} + k\cdot\pi$ $x_3 = -\frac{\pi}{4} + k\cdot\pi$ $x_4 = \frac{\pi}{4} + k\cdot\pi$ $k \in \mathbb{Z}$

$= \frac{2\pi}{3} + k\pi$ $= \frac{3\pi}{4} + k\cdot\pi$

$\frac{\pi}{4} + k\frac{\pi}{2}$

$S = \{\frac{\pi}{4} + k\cdot\frac{\pi}{2} \mid k \in \mathbb{Z}\} \cup \{\frac{\pi}{3} + k\cdot\pi \mid k \in \mathbb{Z}\} \cup \{\frac{2\pi}{3} + k\pi \mid k \in \mathbb{Z}\}$

c) $3\cos^2(\alpha) - 3\cos(\alpha) + 1 = 0$ posons $x = \cos(\alpha)$

$3x^2 - 3x + 1 = 0$

$\Delta = (-3)^2 - 4\cdot 3\cdot 1 = 9 - 12 < 0 \Rightarrow S = \emptyset$

d) $1 - \sin(\alpha) = 2\cos^2(\alpha)$ $\cos^2(\alpha) + \sin^2(\alpha) = 1 \Rightarrow \cos^2(\alpha) = 1 - \sin^2(\alpha)$

$1 - \sin(\alpha) = 2(1 - \sin^2(\alpha))$

$1 - \sin(\alpha) = 2 - 2\sin^2(\alpha)$

$2\sin^2(\alpha) - \sin(\alpha) - 1 = 0$ posons $\sin(\alpha) = x$

$2x^2 - x - 1 = 0 \Rightarrow 2x^2 - 2x + x - 1 = 0 \Rightarrow 2x(x-1) + (x-1) = 0$

$(2x+1)(x-1) = 0$

$2x + 1 = 0$ ou $x - 1 = 0$

$2\cdot\sin(\alpha) + 1 = 0$ $\sin(\alpha) = 1$

$2\sin(\alpha) = -1$ $\alpha_3 = 90° + k\cdot 360°$

$\sin(\alpha) = -\frac{1}{2}$

$S = \{90° + k\cdot 360° \mid k \in \mathbb{Z}\} \cup \{-30° + k\cdot 360° \mid k \in \mathbb{Z}\} \cup \{210° + k\cdot 360° \mid k \in \mathbb{Z}\}$

$\alpha_1 = -30° + k\cdot 360°$

$\alpha_2 = 210° + k\cdot 360°$

Exercice 13

$h(t) = 1{,}2 \cdot \sin(0{,}506 \cdot t + 2) + 5$

a) le sin oscille entre -1 et 1. $1{,}2 \cdot \sin(x)$ oscille entre $-1{,}2$ et $1{,}2$, il y a donc une différence de $2{,}4$ m.

b) la période du $\sin(x)$ est de 2π.

$p = \frac{2\pi}{0{,}506} = 12{,}42$ h $\Rightarrow$ ~12h 25 minutes s'écoule entre 2 marées hautes

c) La hauteur maximale est atteinte lorsque $\sin(x) = 1$.

$\sin(0{,}506 \cdot t + 2) = 1$

$0{,}506 \cdot t + 2 = \frac{\pi}{2} + k \cdot 2\pi$

$0{,}506\, t = \frac{\pi}{2} - 2 + k \cdot 2\pi$

$t = -0{,}8482 + k \cdot 12{,}4174$

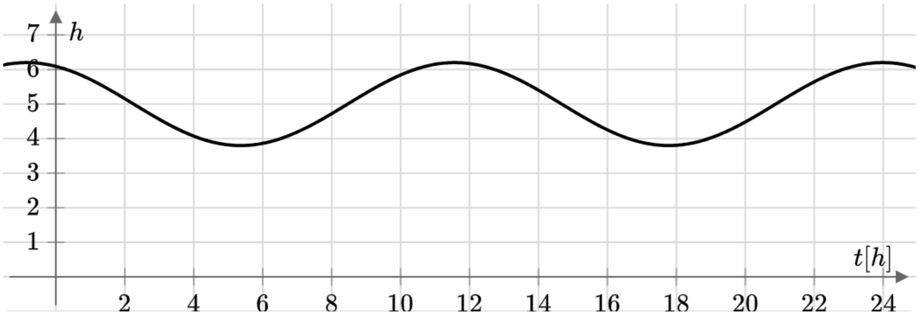
$t = 11{,}5691 \rightarrow \sim 11h34$

La première marée haute a lieu à ~11h34 min.

d) $h(13{,}5) = 1{,}2 \cdot \sin(0{,}506 \cdot 13{,}5 + 2) + 5 = 5{,}67$ m

Le niveau de la mer à 13h30 est de 5,67 m et il décroît, car la 1ère marée haute est à 11h34.

e)



Exercice 14

a) $\sin(x) - \sin(x)\cos(x) = 0$

$S = \{ k \cdot \pi \mid k \in \mathbb{Z} \}$

$\sin(x)(1 - \cos(x)) = 0$

$\sin(x) = 0$ $\quad$ $\cos(x) = 1$

$x_1 = k \cdot \pi$ $\quad$ $x_2 = k \cdot 2\pi$

b) $3\tan(\alpha)\cdot\sin(\alpha)-\sin(\alpha)=0$

$\sin(\alpha)(3\tan(\alpha)-1)=0$

$\sin(\alpha)=0$ $\qquad$ $3\tan(\alpha)-1=0$

$\alpha=k\cdot180°$

$3\tan(\alpha)=1$

$\tan(\alpha)=\frac{1}{3}$

$\alpha=18{,}43°+k\cdot180°$

$S=\{k\cdot180°\mid k\in\mathbb{Z}\}\cup\{18{,}43°+k\cdot180°\mid k\in\mathbb{Z}\}$

c) $\cos\left(3x+\frac{\pi}{6}\right)=0{,}5$

$3x+\frac{\pi}{6}=\frac{\pi}{3}+k\cdot2\pi$

$3x=\frac{\pi}{6}+k\cdot2\pi$

$x_1=\frac{\pi}{18}+k\cdot\frac{2\pi}{3}$

$3x+\frac{\pi}{6}=-\frac{\pi}{3}+k\cdot2\pi$

$3x=-\frac{\pi}{2}+k\cdot2\pi$

$x=-\frac{\pi}{6}+k\cdot\frac{2\pi}{3}$

$S=\left\{\frac{\pi}{18}+k\cdot\frac{2\pi}{3}\mid k\in\mathbb{Z}\right\}\cup\left\{\underbrace{-\frac{\pi}{6}+k\cdot\frac{2\pi}{3}}_{=-\frac{\pi}{2}+k\cdot\frac{2\pi}{3}}\mid k\in\mathbb{Z}\right\}$

d) $\tan\left(2x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$

$2x-\frac{\pi}{6}=\frac{\pi}{6}+k\cdot\pi$

$2x=\frac{\pi}{3}+k\pi$

$x=\frac{\pi}{6}+k\cdot\frac{\pi}{2}$

$S=\left\{\frac{\pi}{6}+k\cdot\frac{\pi}{2}\mid k\in\mathbb{Z}\right\}$

e) $\tan(5x-2)=100$

$5x-2=1{,}5608+k\cdot\pi$

$5x=3{,}5608+k\pi$

$x=0{,}7122+k\cdot\frac{\pi}{5}$

$S=\left\{0{,}7122+k\cdot\frac{\pi}{5}\right\}$

f) $4\cdot\cos^2(\alpha)-3=0$ $\qquad$ pasau $\cos(\alpha)=x$

$4x^2-3=0$

$(2x+\sqrt{3})(2x-\sqrt{3})=0$

$2x+\sqrt{3}=0$ $\qquad$ $2x-\sqrt{3}=0$

$2\cdot\cos(\alpha)=-\sqrt{3}$

$\cos(\alpha)=-\frac{\sqrt{3}}{2}$

$\alpha_1=150°+k\cdot360°$

$\alpha_2=-150°+k\cdot360°$

$2\cos(\alpha)=\sqrt{3}$

$\cos(\alpha)=\frac{\sqrt{3}}{2}$

$\alpha_3=30°+k\cdot360°$

$\alpha_4=-30°+k\cdot360°$

$S=\{30°+k\cdot180°\mid k\in\mathbb{Z}\}\cup\{-30°+k\cdot180°\mid k\in\mathbb{Z}\}$

g) $2\cos^2(x) - \cos(x) - 0{,}5 = 0$ posons $y = \cos(x)$

$2y^2 - y - 0{,}5 = 0$

$\Delta = (-1)^2 - 4\cdot 2\cdot(-0{,}5) = 1 + 4 = 5$

$y_{1,2} = \dfrac{1 \pm \sqrt{5}}{4}$

$\cos(x) = \dfrac{1+\sqrt{5}}{4}$ $\cos(x) = \dfrac{1-\sqrt{5}}{4}$

$x_1 = \dfrac{\pi}{5} + k\cdot 2\pi$ $x_2 = -\dfrac{\pi}{5} + k\cdot 2\pi$ $x_3 = \dfrac{3\pi}{5} + k\cdot 2\pi$ $x_4 = -\dfrac{3\pi}{5} + k\cdot 2\pi$

$S = \left\{ \dfrac{\pi}{5} + k\cdot 2\pi \mid k\in\mathbb{Z} \right\} \cup \left\{ -\dfrac{\pi}{5} + k\cdot 2\pi \mid k\in\mathbb{Z} \right\} \cup \left\{ \dfrac{3\pi}{5} + k\cdot 2\pi \mid k\in\mathbb{Z} \right\} \cup \left\{ -\dfrac{3\pi}{5} + k\cdot 2\pi \mid k\in\mathbb{Z} \right\}$

h) $3\cdot\sin(\alpha) = 2\cdot\cos^2(\alpha)$ $\cos^2(\alpha) = 1 - \sin^2(\alpha)$

$3\sin(\alpha) = 2\cdot(1 - \sin^2(\alpha))$

$3\sin(\alpha) = 2 - 2\sin^2(\alpha)$

$2\sin^2(\alpha) + 3\sin(\alpha) - 2 = 0$ posons $x = \sin(\alpha)$

$2x^2 + 3x - 2 = 0$

$2x^2 + 4x - x - 2 = 0 \Rightarrow 2x(x+2) - (x+2) = 0 \Rightarrow (2x-1)(x+2) = 0$

$2x - 1 = 0 \Rightarrow 2\sin(\alpha) - 1 = 0$ | $\sin(\alpha) + 2 = 0 \Rightarrow \sin(\alpha) = -2$ impossible

$\sin(\alpha) = \dfrac{1}{2} \Rightarrow \alpha_1 = 30^\circ + k\cdot 360^\circ$, $\alpha_2 = 150^\circ + k\cdot 360^\circ$ | $S = \{30^\circ + k\cdot 360^\circ \mid k\in\mathbb{Z}\} \cup \{150^\circ + k\cdot 360^\circ \mid k\in\mathbb{Z}\}$

i) $\tan(\alpha) - \sin(\alpha) = 0$ $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\dfrac{\sin(\alpha)}{\cos(\alpha)} - \sin(\alpha) = 0$

$\dfrac{\sin(\alpha) - \sin(\alpha)\cos(\alpha)}{\cos(\alpha)} = 0 \Rightarrow \alpha \neq 90^\circ$ et 270°

$\sin(\alpha) - \sin(\alpha)\cos(\alpha) = 0$

$\sin(\alpha)(1 - \cos(\alpha)) = 0$

$\sin(\alpha) = 0$ $1 - \cos(\alpha) = 0$

$1 = \cos(\alpha)$

$\alpha = k\cdot 360^\circ$

$\alpha = k\cdot 180^\circ$

$S = \{k\cdot 180^\circ \mid k\in\mathbb{Z}\}$

j) $\sin(x) + \cos(x) = 1 \quad \Big| \cdot \frac{\sqrt{2}}{2}$

$$\frac{\sqrt{2}}{2}\sin(x) + \frac{\sqrt{2}}{2}\cos(x) = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)$$

$$\cos\left(\frac{\pi}{4}\right)\sin(x) + \cos\left(\frac{\pi}{4}\right)\cos(x) = \frac{\sqrt{2}}{2}$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$\arcsin(\ldots)$

$$x + \frac{\pi}{4} = \frac{\pi}{4} + k \cdot 2\pi \qquad\qquad x + \frac{\pi}{4} = \pi - \frac{\pi}{4} + k \cdot 2\pi$$

$$x = k \cdot 2\pi \qquad\qquad x + \frac{\pi}{4} = \frac{3\pi}{4} + k \cdot 2\pi$$

$$x = \frac{\pi}{2} + k \cdot 2\pi$$

$$S = \{ k \cdot 2\pi \mid k \in \mathbb{Z} \} \cup \{ \frac{\pi}{2} + k \cdot 2\pi \mid k \in \mathbb{Z} \}$$