

## Vecteurs - solutions détaillées

### Exercice 1

$$2 \cdot \vec{f} = \vec{\ell} \quad -\frac{1}{9} \cdot \vec{f} = \vec{0} \quad \Rightarrow \quad \vec{f}, \vec{\ell} \text{ et } \vec{0} \text{ sont colinéaires.}$$

$$\vec{g} \cdot \left(-\frac{2}{3}\right) = \vec{p} \quad \Rightarrow \quad \vec{g} \text{ et } \vec{p} \text{ sont colinéaires.}$$

$$-2 \cdot \vec{n} = \vec{h} \quad \Rightarrow \quad \vec{h} \text{ et } \vec{n} \text{ sont colinéaires.}$$

### Exercice 2

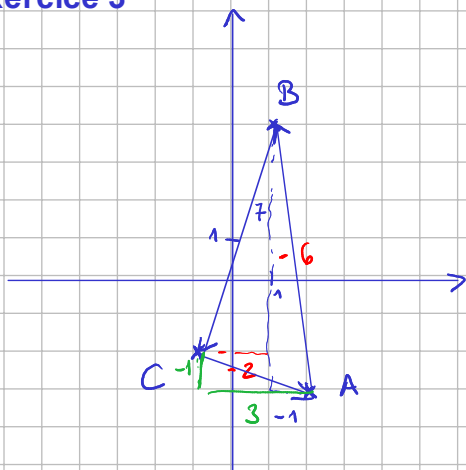
$$a) \quad \vec{AB} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 1-6 \\ 5+2 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$$

$$b) \quad \vec{AB} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -3-2 \\ -3-7 \end{pmatrix} = \begin{pmatrix} -5 \\ -10 \end{pmatrix}$$

$$c) \quad \vec{AB} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 5+4 \\ 5-1 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$$d) \quad \vec{AB} = \begin{pmatrix} 10.5 \\ 5.5 \end{pmatrix} - \begin{pmatrix} 4.5 \\ 6.5 \end{pmatrix} = \begin{pmatrix} 10.5-4.5 \\ 5.5-6.5 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

### Exercice 3



$$a) \quad \vec{AB} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

$$b) \quad \vec{BC} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$$

$$c) \quad \vec{CA} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$d) \quad \vec{BA} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$$

### Exercice 4

P, Q et R sont alignés si  $\vec{PQ}$  et  $\vec{QR}$  sont colinéaires.

$$a) \quad \vec{PQ} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad \vec{QR} = \begin{pmatrix} 7 \\ 8 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \end{pmatrix}$$

$$\det(\vec{PQ}, \vec{QR}) = \begin{vmatrix} 1 & 1 \\ -5 & 11 \end{vmatrix} = 1 \cdot 11 - (-5) \cdot 1 = 11 + 5 = 16 \neq 0$$

$\Rightarrow$  P, Q et R ne sont pas alignés.

#### Exercice 4 (suite)

$$b) \vec{PQ} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ -6 \end{pmatrix} = \begin{pmatrix} -2 \\ 14 \end{pmatrix} \quad \vec{QR} = \begin{pmatrix} -1 \\ 36 \end{pmatrix} - \begin{pmatrix} 3 \\ 8 \end{pmatrix} = \begin{pmatrix} -4 \\ 28 \end{pmatrix}$$

$$\det(\vec{PQ}; \vec{QR}) = \begin{vmatrix} -2 & -4 \\ 14 & 28 \end{vmatrix} = -2 \cdot 28 - 14 \cdot (-4) = -56 + 56 = 0$$

$\Rightarrow P, Q$  et  $R$  sont alignés.

#### Exercice 5

$$\vec{AB} = \begin{pmatrix} -8 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -9 \\ 9 \end{pmatrix}$$

$$\|\vec{AB}\| = \sqrt{(-9)^2 + 9^2} = \sqrt{2 \cdot 81} = 9\sqrt{2}$$

$$\vec{BC} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -8 \\ 12 \end{pmatrix} = \begin{pmatrix} 9 \\ -10 \end{pmatrix}$$

$$\|\vec{BC}\| = \sqrt{9^2 + 10^2} = \sqrt{81 + 100} = \sqrt{181}$$

$$\vec{CA} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\|\vec{CA}\| = \sqrt{0^2 + 1^2} = 1$$

Périmètre:  $\|\vec{AB}\| + \|\vec{BC}\| + \|\vec{CA}\| = 9\sqrt{2} + \sqrt{181} + 1 \approx 27,18$

#### Exercice 6

$$\vec{AB} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

$$\|\vec{AB}\| = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{2 \cdot 9} = 3\sqrt{2}$$

$$\vec{BC} = \begin{pmatrix} \frac{17}{2} \\ \frac{9}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ \frac{11}{2} \end{pmatrix}$$

$$\|\vec{BC}\| = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{11}{2}\right)^2} = \sqrt{\frac{25}{4} + \frac{121}{4}} = \sqrt{\frac{146}{4}} = \frac{\sqrt{146}}{2}$$

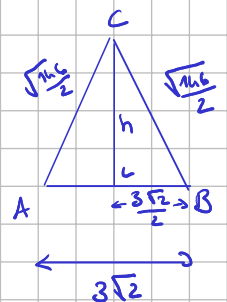
$$\vec{CA} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{17}{2} \\ \frac{9}{2} \end{pmatrix} = \begin{pmatrix} -\frac{11}{2} \\ -\frac{5}{2} \end{pmatrix}$$

$$\|\vec{CA}\| = \sqrt{\left(-\frac{11}{2}\right)^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{\frac{121}{4} + \frac{25}{4}} = \sqrt{\frac{146}{4}} = \frac{\sqrt{146}}{2}$$

Le triangle est isocèle en C.

Par Pythagore:  $h = \sqrt{\left(\frac{\sqrt{146}}{2}\right)^2 - \left(\frac{3\sqrt{2}}{2}\right)^2} = \sqrt{\frac{146}{4} - \frac{9 \cdot 2}{4}} = \sqrt{\frac{146-18}{4}} = \sqrt{\frac{128}{4}} = \sqrt{32} = \sqrt{2 \cdot 16} = 4\sqrt{2}$

Aire:  $\frac{3\sqrt{2} \cdot 4\sqrt{2}}{2} = \frac{12 \cdot 2}{2} = 12$



### Exercise 7

Un vecteur est unitaire  $\Leftrightarrow$  sa norme vaut 1.

$$a) \vec{v} = \begin{pmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{pmatrix} \quad \|\vec{v}\| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$

$$b) \vec{v} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad \|\vec{v}\| = \sqrt{(-1)^2 + 0^2} = \sqrt{1+0} = \sqrt{1} = 1$$

$$c) \vec{v} = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad \|\vec{v}\| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = \sqrt{1} = 1$$

### Exercise 8

$$a) \vec{a} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \|\vec{a}\| = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$a' = \pm \frac{\vec{a}}{\|\vec{a}\|} = \pm \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \pm \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$b) \vec{b} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \quad \|\vec{b}\| = \sqrt{(-5)^2 + 0^2} = \sqrt{25+0} = \sqrt{25} = 5$$

$$b' = \pm \frac{\vec{b}}{\|\vec{b}\|} = \pm \frac{1}{5} \begin{pmatrix} -5 \\ 0 \end{pmatrix} = \pm \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$c) \vec{c} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad \|\vec{c}\| = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$c' = \pm \frac{1}{5} \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \pm \begin{pmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{pmatrix}$$

### Exercise 9

$$a) \vec{v} = 5\vec{v}_1 - 2\vec{v}_2 = 5 \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 15 \end{pmatrix} + \begin{pmatrix} -10 \\ -6 \end{pmatrix} = \begin{pmatrix} -5-10 \\ 15-6 \end{pmatrix} = \begin{pmatrix} -15 \\ 9 \end{pmatrix}$$

$$b) \frac{5}{5} \vec{v} = \vec{v}_3 - 2\vec{v}_2 - \frac{3}{2} \vec{v}_4 \quad \cdot \frac{3}{5}$$

$$\vec{v} = \frac{3}{5} \left( \vec{v}_3 - 2\vec{v}_2 - \frac{3}{2} \vec{v}_4 \right)$$

$$\vec{v} = \frac{3}{5} \vec{v}_3 - \frac{6}{5} \vec{v}_2 - \frac{9}{10} \vec{v}_4 = \frac{3}{5} \cdot \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \frac{6}{5} \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \frac{9}{10} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{6}{5} \end{pmatrix} + \begin{pmatrix} -6 \\ -\frac{18}{5} \end{pmatrix} + \begin{pmatrix} -\frac{27}{10} \\ -\frac{18}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 0-6-\frac{27}{10} \\ -\frac{6}{5}-\frac{18}{5}-\frac{18}{5} \end{pmatrix} = \begin{pmatrix} \frac{-60-27}{10} \\ \frac{-6-18-18}{5} \end{pmatrix} = \begin{pmatrix} -\frac{87}{10} \\ -\frac{42}{5} \end{pmatrix}$$

### Exercise 10

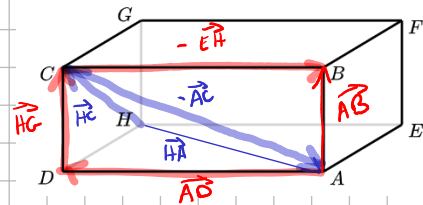
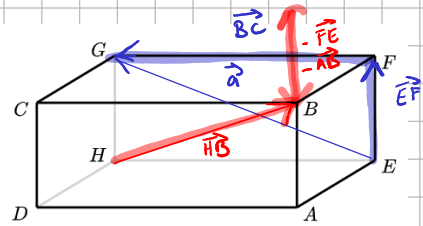
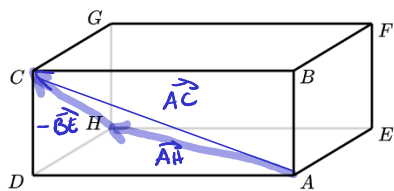
$$a) \vec{a} = \vec{EF} + \vec{BC} = \vec{EG} = \vec{AC}$$

$$b) \vec{b} = \vec{HB} - \vec{FE} - \vec{AB} = \vec{HB}$$

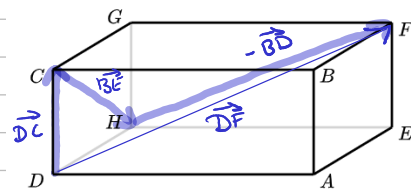
$$c) \vec{c} = \vec{HC} - \vec{AC} = \vec{HA}$$

$$d) \vec{d} = \vec{AB} + \vec{HG} - \vec{EH} = \vec{AB}$$

$$e) \vec{e} = \vec{AH} - \vec{BE} = \vec{AC}$$



$$f) \vec{f} = \vec{DC} + \vec{BE} - \vec{BD} + \vec{FE} = \vec{DF}$$



### Exercise 11

$$a) \vec{BE} = \vec{BD} + \vec{DE}$$

$$b) \vec{BF} = \frac{1}{2} \vec{BA} - \vec{AD}$$

$$c) \vec{DG} = 2 \vec{DC} + \frac{1}{2} \vec{CA}$$

$$d) \vec{AH} = -\sqrt{2} \vec{AC}$$

