Produit scalaire, angle de deux vecteurs - corrigés détaillés

Exercice 1

a) $\vec{v} \cdot \vec{w} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \cdot 1 + (-1) \cdot 1 = 0 \quad \Rightarrow \quad \vec{v} \perp \vec{w} \quad \Rightarrow \quad \varphi = 90°$

$\left(\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos(\varphi) = \sqrt{1^2 + (-1)^2} \cdot \sqrt{1^2 + 1^2} \cdot \cos(\varphi) = \sqrt{2} \cdot \sqrt{2} \cdot \cos(\varphi) = 2\cos(\varphi) \right.$

$\left. \Rightarrow \quad 0 = 2 \cdot \cos(\varphi) \quad \Rightarrow \quad \cos(\varphi) = 0 \quad \Rightarrow \quad \varphi = \cos^{-1}(\varphi) = 90° \right)$

b) $\vec{v} \cdot \vec{w} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 2 \cdot 1 + 1 \cdot 2 = 2 + 2 = 4$

$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos(\varphi) = \sqrt{2^2 + 1^2} \cdot \sqrt{1^2 + 2^2} \cdot \cos(\varphi) = \sqrt{5} \cdot \sqrt{5} \cdot \cos(\varphi) = 5\cos(\varphi)$

$\Rightarrow \quad 4 = 5\cos(\varphi) \quad \Rightarrow \quad \cos(\varphi) = \frac{4}{5} \quad \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{4}{5}\right) = 36{,}87°$

c) $\vec{v} \cdot \vec{w} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix} = 1 \cdot 4 + (-1) \cdot (-3) = 4 + 3 = 7$

$\vec{v} \cdot \vec{w} = \sqrt{1^2 + (-1)^2} \cdot \sqrt{4^2 + (-3)^2} \cdot \cos(\varphi) = \sqrt{2} \cdot \sqrt{25} \cdot \cos(\varphi) = 5\sqrt{2} \cdot \cos(\varphi)$

$\Rightarrow \quad 7 = 5\sqrt{2} \cdot \cos(\varphi) \quad \Rightarrow \quad \cos(\varphi) = \frac{7}{5\sqrt{2}} \quad \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{7}{5\sqrt{2}}\right) = 8{,}13°$

d) $\vec{v} \cdot \vec{w} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 2 \cdot 1 + 2 \cdot 2 = 2 + 4 = 6$

$\vec{v} \cdot \vec{w} = \sqrt{2^2 + 2^2} \cdot \sqrt{1^2 + 2^2} \cdot \cos(\varphi) = \sqrt{8} \cdot \sqrt{5} \cdot \cos(\varphi) = 2\sqrt{10} \cdot \cos(\varphi)$

$\Rightarrow \quad 6 = 2\sqrt{10} \cdot \cos(\varphi) \quad \Rightarrow \quad \cos(\varphi) = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}} \quad \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right) = 18{,}43°$

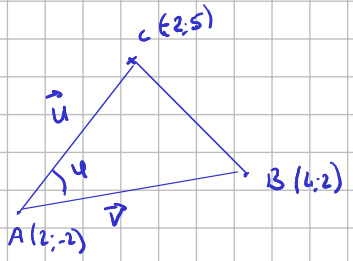
e) $\vec{v} \cdot \vec{w} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 3 \cdot 1 + (-4) \cdot 1 = 3 - 4 = -1$

$\vec{v} \cdot \vec{w} = \sqrt{3^2 + (-4)^2} \cdot \sqrt{1^2 + 1^2} \cdot \cos(\varphi) = \sqrt{25} \cdot \sqrt{2} \cdot \cos(\varphi) = 5\sqrt{2} \cdot \cos(\varphi)$

$\Rightarrow \quad -1 = 5\sqrt{2} \cdot \cos(\varphi) \quad \Rightarrow \quad \cos(\varphi) = \frac{-1}{5\sqrt{2}} \quad \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{-1}{5\sqrt{2}}\right) = 98{,}13°$

Exercie 2

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos(\gamma) \Rightarrow \cos(\gamma) = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$



$\vec{AC} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$ $\quad \vec{CB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\cos(\alpha) = \dfrac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \cdot \|\vec{AC}\|} = \dfrac{2 \cdot (-4) + 4 \cdot 7}{\sqrt{2^2+4^2} \cdot \sqrt{(-4)^2+7^2}} = \dfrac{-8+28}{\sqrt{20} \cdot \sqrt{65}} = \dfrac{20}{10\sqrt{13}} = \dfrac{2}{\sqrt{13}}$

$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{2}{\sqrt{13}}\right) = 56{,}31°$

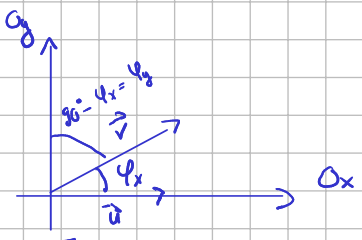
$\cos(\beta) = \dfrac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \dfrac{(-2) \cdot (-6) + (-4) \cdot 3}{\sqrt{(-2)^2+4^2} \cdot \sqrt{(-6)^2+3^2}} = \dfrac{12-12}{\sqrt{20} \cdot \sqrt{45}} = 0 \quad \Rightarrow \beta = \cos^{-1}(0) = 90°$

$\cos(\gamma) = \dfrac{\vec{CA} \cdot \vec{CB}}{\|\vec{CA}\| \cdot \|\vec{CB}\|} = \dfrac{4 \cdot 6 + (-7) \cdot (-3)}{\sqrt{4^2+(-7)^2} \cdot \sqrt{6^2+(-3)^2}} = \dfrac{24+21}{\sqrt{65} \cdot \sqrt{45}} = \dfrac{45}{15\sqrt{13}} = \dfrac{3}{\sqrt{13}}$

$\Rightarrow \gamma = \cos^{-1}\left(\dfrac{3}{\sqrt{13}}\right) = 33{,}69°$

Exercice 3

$\vec{v} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ On choisit un vecteur sur Ox par exemple $\vec{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



$\cos(\varphi_x) = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|} = \dfrac{1 \cdot 4 + 0 \cdot (-2)}{\sqrt{1^2+0^2} \cdot \sqrt{4^2+(-2)^2}} = \dfrac{4}{1 \cdot \sqrt{20}} = \dfrac{4}{2\sqrt{5}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

$\varphi_x = \cos^{-1}\left(\dfrac{2\sqrt{5}}{5}\right) = 26{,}57°$ $\qquad \varphi_y = 180° - 26{,}57° = 63{,}43°$

Exercice 4

$\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$

a) $\vec{u} \cdot \vec{v} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} m \\ 2 \end{pmatrix} = -5m + 6 \cdot 2 = -5m + 12$

$-5m + 12 = 0 \Rightarrow 5m = 12 \Rightarrow m = \dfrac{12}{5}$ $\qquad \vec{v} = \begin{pmatrix} \frac{12}{5} \\ 2 \end{pmatrix}$

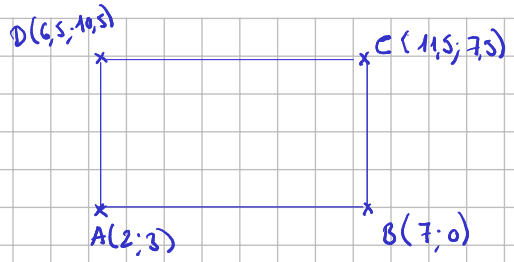
b) $\vec{u} \cdot \vec{w} = \begin{pmatrix} m-6 \\ 4m \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3m-1 \end{pmatrix} = (m-6) \cdot 2 + 4m(3m-1) = 2m - 12 + 12m^2 - 4m = 12m^2 - 2m - 12$

$12m^2 - 2m - 12 = 0 \Rightarrow 6m^2 - m - 6 = 0$

$\Delta = (-1)^2 - 4 \cdot 6 \cdot (-6) = 145$

$m_{1,2} = \dfrac{1 \pm \sqrt{145}}{2 \cdot 6} = \dfrac{1 \pm \sqrt{145}}{12}$

Exercice 5



ABCD est un rectangle si

1) $\vec{AB} = \vec{DC}$ et $\vec{AD} = \vec{BC}$

2) un angle est droit

1) $\vec{AB} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ $\quad \vec{DC} = \begin{pmatrix} 11,5 \\ 7,5 \end{pmatrix} - \begin{pmatrix} 6,5 \\ 10,5 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ $\Rightarrow \vec{AB} = \vec{DC}$

$\vec{AD} = \begin{pmatrix} 6,5 \\ 10,5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4,5 \\ 7,5 \end{pmatrix}$ $\quad \vec{BC} = \begin{pmatrix} 11,5 \\ 7,5 \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \end{pmatrix} = \begin{pmatrix} 4,5 \\ 7,5 \end{pmatrix}$ $\Rightarrow \vec{AD} = \vec{BC}$

2) $\vec{AB} \perp \vec{AD} \iff \vec{AB} \cdot \vec{AD} = 0$

$\begin{pmatrix} 5 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 4,5 \\ 7,5 \end{pmatrix} = 5 \cdot 4,5 + (-3) \cdot 7,5 = 0$

$\Rightarrow$ ABCD est un rectangle.

Exercice 6

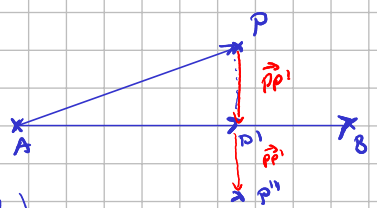
$\vec{u} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ $\qquad \vec{v} \perp \vec{u} \iff \vec{v} = \lambda \cdot \begin{pmatrix} -u_2 \\ u_1 \end{pmatrix} = \lambda \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ $\qquad \lambda \in \mathbb{R}^*$

$\| \vec{v} \| = \sqrt{(\lambda 12)^2 + (5\lambda)^2} = \sqrt{144\lambda^2 + 25\lambda^2} = \sqrt{169\lambda^2} = \pm 13\lambda$

$\| \vec{v} \| = \frac{13}{2} \Rightarrow \pm 13\lambda = \frac{13}{2} \Rightarrow \lambda = \pm \frac{1}{2} \Rightarrow \vec{v} = \pm \frac{1}{2} \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \pm \begin{pmatrix} 6 \\ \frac{5}{2} \end{pmatrix}$

Exercice 7

$\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\| \vec{v} \|^2} \cdot \vec{v} \Rightarrow \vec{AP'} = \frac{\vec{AP} \cdot \vec{AB}}{\| \vec{AB} \|^2} \cdot \vec{AB}$



a) A(1;3), B(5;-1) et P(2;6)

$\vec{AP} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\qquad \vec{AB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

$\vec{AP'} = \frac{1 \cdot 4 + 3 \cdot (-4)}{(\sqrt{4^2 + (-4)^2})^2} \cdot \vec{AB} = \frac{4 - 12}{(\sqrt{32})^2} \cdot \vec{AB} = -\frac{8}{32} \cdot \vec{AB} = -\frac{1}{4} \cdot \begin{pmatrix} 4 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\vec{OP'} = \vec{OA} + \vec{AP'} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \Rightarrow P'(0;4)$

$\vec{PP'} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\vec{OP''} = \vec{OP'} + \vec{PP'} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \Rightarrow P''(-2;2)$

Exercice 7 (suite)

b) $A(1;2)$ $B(3;6)$ $P(2;1)$

$\vec{AP} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\qquad \vec{AB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\vec{AP'} = \frac{1 \cdot 2 + (-1) \cdot 4}{(\sqrt{2^2+4^2})^2} \cdot \vec{AB} = \frac{2-4}{(\sqrt{20})^2} \cdot \vec{AB} = \frac{-2}{20} \cdot \vec{AB} = -\frac{1}{10} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{5} \\ -\frac{2}{5} \end{pmatrix}$

$\vec{OP'} = \vec{OA} + \vec{AP'} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{5} \\ -\frac{2}{5} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{5} \\ 2-\frac{2}{5} \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{8}{5} \end{pmatrix} \Rightarrow P'\left(\frac{4}{5}; \frac{8}{5}\right)$

$\vec{PP'} = \begin{pmatrix} \frac{4}{5} \\ \frac{8}{5} \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{6}{5} \\ \frac{3}{5} \end{pmatrix}$

$\vec{OP''} = \vec{OP'} + \vec{PP'} = \begin{pmatrix} \frac{4}{5} \\ \frac{8}{5} \end{pmatrix} + \begin{pmatrix} -\frac{6}{5} \\ \frac{3}{5} \end{pmatrix} = \begin{pmatrix} -\frac{2}{5} \\ \frac{11}{5} \end{pmatrix} \Rightarrow P''\left(-\frac{2}{5}; \frac{11}{5}\right)$

c) $A(10;-2)$ $B(3;5)$ $P(-4;3)$

$\vec{AP} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 10 \\ -2 \end{pmatrix} = \begin{pmatrix} -14 \\ 5 \end{pmatrix}$ $\qquad \vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 10 \\ -2 \end{pmatrix} = \begin{pmatrix} -7 \\ 7 \end{pmatrix}$

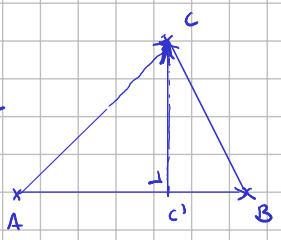
$\vec{AP'} = \frac{(-14) \cdot (-7) + 5 \cdot 7}{(\sqrt{(-7)^2 + 7^2})^2} \cdot \vec{AB} = \frac{98+35}{(\sqrt{98})^2} \cdot \vec{AB} = \frac{133}{98} \cdot \vec{AB} = \frac{19}{14} \begin{pmatrix} -7 \\ 7 \end{pmatrix} = \begin{pmatrix} -\frac{19}{2} \\ \frac{19}{2} \end{pmatrix}$

$\vec{OP'} = \vec{OA} + \vec{AP'} = \begin{pmatrix} 10 \\ -2 \end{pmatrix} + \begin{pmatrix} -\frac{19}{2} \\ \frac{19}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{15}{2} \end{pmatrix} \Rightarrow P'\left(\frac{1}{2}; \frac{15}{2}\right)$

$\vec{PP'} = \begin{pmatrix} \frac{1}{2} \\ \frac{15}{2} \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ \frac{9}{2} \end{pmatrix}$

$\vec{OP''} = \vec{OP'} + \vec{PP'} = \begin{pmatrix} \frac{1}{2} \\ \frac{15}{2} \end{pmatrix} + \begin{pmatrix} \frac{9}{2} \\ \frac{9}{2} \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \Rightarrow P''(5;12)$

Exercice 8



Aire : $\dfrac{\|\vec{AB}\| \cdot \|\vec{CC'}\|}{2}$ où C' est la projection orthogonale de C sur $\vec{AB}$

a) A(3 ; -2) B(-1 ; 2) C(7 ; 5)

$\vec{AB} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\vec{AC'} = \dfrac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\|^2} \cdot \vec{AB} = \dfrac{(-4)\cdot 4 + 4 \cdot 7}{\left(\sqrt{(-4)^2 + 4^2}\right)^2} \cdot \vec{AB} = \dfrac{-16 + 28}{(\sqrt{32})^2} = \dfrac{12}{32} \cdot \vec{AB} = \dfrac{3}{8} \cdot \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \end{pmatrix}$

$\vec{OC'} = \vec{OA} + \vec{AC'} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}$

$\vec{CC'} = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} -\frac{11}{2} \\ -\frac{11}{2} \end{pmatrix}$

Aire : $\dfrac{\sqrt{(-4)^2 + 4^2} \cdot \sqrt{\left(-\frac{11}{2}\right)^2 + \left(-\frac{11}{2}\right)^2}}{2} = \dfrac{\sqrt{32} \cdot \sqrt{\frac{121}{2}}}{2} = \dfrac{\sqrt{\frac{32 \cdot 121}{2}}}{2} = \dfrac{\sqrt{16} \cdot \sqrt{121}}{2} = \dfrac{4 \cdot 11}{2} = 22$

b) $A\left(\frac{9}{2} ; 3\right)$ $B\left(6 ; \frac{21}{2}\right)$ $C\left(12 ; \frac{33}{2}\right)$

$\vec{AB} = \begin{pmatrix} 6 \\ \frac{21}{2} \end{pmatrix} - \begin{pmatrix} \frac{9}{2} \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix} 12 \\ \frac{33}{2} \end{pmatrix} - \begin{pmatrix} \frac{9}{2} \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{27}{2} \end{pmatrix}$

$\vec{AC'} = \dfrac{\frac{3}{2} \cdot \frac{15}{2} + \frac{15}{2} \cdot \frac{27}{2}}{\left(\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{15}{2}\right)^2}\right)^2} \cdot \vec{AB} = \dfrac{\frac{45}{4} + \frac{405}{4}}{\left(\sqrt{\frac{9}{4} + \frac{225}{4}}\right)^2} \cdot \vec{AB} = \dfrac{\frac{450}{4}}{\left(\sqrt{\frac{234}{4}}\right)^2} \cdot \vec{AB} = \dfrac{225}{2} \cdot \dfrac{4}{234} \cdot \vec{AB}$

$= \dfrac{25}{13} \cdot \vec{AB} = \dfrac{25}{13} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{75}{26} \\ \frac{375}{26} \end{pmatrix}$

$\vec{OC'} = \vec{OA} + \vec{AC'} = \begin{pmatrix} \frac{9}{2} \\ 3 \end{pmatrix} + \begin{pmatrix} \frac{75}{26} \\ \frac{375}{26} \end{pmatrix} = \begin{pmatrix} \frac{96}{13} \\ \frac{453}{26} \end{pmatrix}$

$\vec{CC'} = \begin{pmatrix} \frac{96}{13} \\ \frac{453}{26} \end{pmatrix} - \begin{pmatrix} 12 \\ \frac{33}{2} \end{pmatrix} = \begin{pmatrix} -\frac{60}{13} \\ \frac{12}{13} \end{pmatrix}$

Aire : $\dfrac{\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{15}{2}\right)^2} \cdot \sqrt{\left(-\frac{60}{13}\right)^2 + \left(\frac{12}{13}\right)^2}}{2} = \dfrac{\sqrt{\frac{117}{2}} \cdot \sqrt{\frac{288}{13}}}{2} = \dfrac{36}{2} = 18$

Exercice 9

a) $d: 3x + 2y - 1 = 0$

$2y = -3x + 1$

$y = -\frac{3}{2}x + \frac{1}{2}$

$d' \perp d \iff m_{d'} \cdot m_d = -1 \Rightarrow m_{d'} = \frac{-1}{m_d}$

$m_{d'} = \frac{-1}{-\frac{3}{2}} = \frac{2}{3}$

$d': y = \frac{2}{3}x + h$

$P(4;2) \in d' \Rightarrow 2 = \frac{2}{3} \cdot 4 + h \Rightarrow h = 2 - \frac{8}{3} = \frac{6-8}{3} = -\frac{2}{3}$

$d': y = \frac{2}{3}x - \frac{2}{3} \quad | \cdot 3$

$3y = 2x - 2$

$0 = 2x - 3y - 2 \Rightarrow d': 2x - 3y - 2 = 0$

b) $d: y = -2x + 4 \Rightarrow m_{d'} = \frac{-1}{-2} = \frac{1}{2}$

$d': y = \frac{1}{2}x + h$

$P(7;2) \in d' \Rightarrow 2 = \frac{1}{2} \cdot 7 + h \Rightarrow h = 2 - \frac{7}{2} = \frac{4-7}{2} = -\frac{3}{2}$

$d': y = \frac{1}{2}x - \frac{3}{2} \quad | \cdot 2$

$2y = x - 3$

$0 = x - 2y - 3 \Rightarrow d': x - 2y - 3 = 0$

c) $d: x = 2$ d' sera une droite constante $d': y = h$

$P(-1;1) \in d' \Rightarrow 1 = h$

$d': y = 1$

d) $d: y = 3x - 8 \quad m_{d'} = -\frac{1}{3}$

$d': y = -\frac{1}{3}x + h$

$P(6;-7) \in d' \Rightarrow -7 = -\frac{1}{3} \cdot 6 + h \Rightarrow h = -7 + 2 = -5$

$d': y = -\frac{1}{3}x - 5 \quad | \cdot 3$

$3y = -x - 15$

$d': x + 3y + 15 = 0$