Fonctions révisions - Solutions détaillées

Exercice 3

Le dénominateur ne doit jamais être nul.

La partie sous la racine doit être positive ou nulle.

a) $f(x) = \dfrac{1}{x^2-4}$ $\quad x^2-4 \neq 0 \iff (x-2)(x+2) \neq 0 \iff x \neq 2$ et $x \neq -2$

$D_f = \mathbb{R} \setminus \{-2; 2\}$

b) $f(x) = \sqrt{x+4}$ $\quad x+4 \geq 0 \iff x \geq -4$ $\quad -4$

$D_f = [-4; +\infty[$

c) $f(x) = \dfrac{2x+4}{1+x^2}$ $\quad 1+x^2 \neq 0$ $\quad$ C'est toujours vrai $\Rightarrow D_f = \mathbb{R}$

d) $f(x) = \dfrac{4x+7}{x}$ $\quad x \neq 0$ $\quad D_f = \mathbb{R} \setminus \{0\} = \mathbb{R}^*$

e) $f(x) = \sqrt{x^2+6x+5}$ $\quad x^2+6x+5 \geq 0$ $\quad$ inéquation du 2e degré $\Rightarrow$ tableau de signes

$(x+5)(x+1) \geq 0$

$x = -5 \qquad x = -1$

x	$-\infty$		-5		-1		$+\infty$
$x+5$		$-$	0	$+$		$+$	
$x+1$		$-$		$-$	0	$+$	
$(x+5)(x+1) \geq 0$		$+$	0	$-$	0	$+$	

$D_f =]-\infty; -5] \cup [-1; +\infty[$

f) $f(x) = \sqrt{6x-4}$ $\quad 6x-4 \geq 0 \iff 6x \geq 4 \iff x \geq \dfrac{4}{6} \iff x \geq \dfrac{2}{3}$ $\quad \frac{2}{3}$

$D_f = [\frac{2}{3}, +\infty[$

g) $f(x) = \sqrt{5-3x}$

$5-3x \geq 0 \quad | -5$

$-3x \geq -5 \quad | :(-3)$

$x \leq \dfrac{5}{3}$

$\frac{5}{3}$

$D_f =]-\infty; \frac{5}{3}]$

Exercice 4

a) $f(x) = 5x - 1$

$D_f = \mathbb{R}$

zéro: $5x - 1 = 0 \Rightarrow x = \frac{1}{5}$

x	$-\infty$		$\frac{1}{5}$		$+\infty$
$5x-1$		$-$	0	$+$	

b) $f(t) = t^2 + 2t = t(t+2)$

$D_f = \mathbb{R}$

zéros: $t_1 = 0$ et $t_2 = -2$

t	$-\infty$		-2		0		$+\infty$
t		$-$		$-$	0	$+$	
$t+2$		$-$	0	$+$		$+$	
$t(t+2)$		$+$	0	$-$	0	$+$	

c) $g(x) = \dfrac{x-4}{4-2x}$

Domaine de définition:

$4 - 2x \neq 0 \iff 4 \neq 2x$

$\iff x \neq 2 \Rightarrow D_g = \mathbb{R} \setminus \{2\}$

zero: $x - 4 = 0 \iff x = 4$

x	$-\infty$		2		4		$+\infty$
$x-4$		$-$		$-$	0	$+$	
$4-2x$		$+$	0	$-$		$-$	
$\dfrac{x-4}{4-2x}$		$-$		$+$	0	$-$	

d) $h(s) = s^2 - 4s + 3 = (s-3)(s-1)$

$D_h = \mathbb{R}$

zéros: $s_1 = 3$ et $s_2 = 1$

s	$-\infty$		1		3		$+\infty$
$s-3$		$-$		$-$	0	$+$	
$s-1$		$-$	0	$+$		$+$	
$(s-3)(s-1)$		$+$	0	$-$	0	$+$	

e) $j(x) = \dfrac{x+3}{2x-4}$

Domaine de définition:

$2x - 4 \neq 0 \iff 2x \neq 4$

$\iff x \neq 2 \Rightarrow D_j = \mathbb{R} \setminus \{2\}$

zéro: $x + 3 = 0 \iff x = -3$

x	$-\infty$		-3		2		$+\infty$
$x+3$		$-$	0	$+$		$+$	
$2x-4$		$-$		$-$	0	$+$	
$\dfrac{x+3}{2x-4}$		$+$	0	$-$		$+$	

f) $k(t)=\sqrt{(t+2)(3-t)}$

$D_k = [-2;3]$

zéros: $t_1=-2$ et $t_2=3$

t	$-\infty$		-2		3		$+\infty$
$t+2$		$-$	0	$+$		$+$	
$3-t$		$+$		$+$	0	$-$	
$(t+2)(3-t)$		$-$	0	$+$	0	$-$	
$\sqrt{(t+2)(3-t)}$		///	0	$+$	0	///	

g) $m(t)=\dfrac{t^2-3t-4}{t}=\dfrac{(t-4)(t+1)}{t}$

$D_m=\mathbb{R}\setminus\{0\}=\mathbb{R}^*$

zéros: $t_1=-1$ et $t_2=4$

t	$-\infty$		-1		0		4		$+\infty$
$t-4$		$-$		$-$	///	$-$	0	$+$	
$t+1$		$-$	0	$+$	///	$+$		$+$	
t		$-$		$-$	0	$+$		$+$	
$\dfrac{(t-4)(t+1)}{t}$		$-$	0	$+$	///	$-$	0	$+$	

h) $f(x)=\dfrac{\sqrt{-x+2}}{4x^2+2x}=\dfrac{\sqrt{-x+2}}{2x(2x+1)}$

$D_f=]-\infty;2]\setminus\{-\frac{1}{2};0\}$

Zéro: $x=2$

x	$-\infty$		$-\frac{1}{2}$		0		2		$+\infty$
$\sqrt{-x+2}$		$+$		$+$		$+$	0	///	
$2x$		$-$		$-$	0	$+$		$+$	
$2x+1$		$-$	0	$+$		$+$		$+$	
$\dfrac{\sqrt{-x+2}}{2x(2x+1)}$		$+$	///	$-$	///	$+$	0	///	

Exercice 6

a) $f(x)=mx+p$

$m=\dfrac{6-4}{3-2}=\dfrac{2}{1}=2 \Rightarrow f(x)=2x+p$

$A(2;4)\in f \Rightarrow f(2)=4$

$2\cdot 2+p=4 \Rightarrow p=0 \Rightarrow f(x)=2x$

b) $f(x)=mx+p$

$m=\dfrac{-1-(-1)}{-2-2}=\dfrac{-1+1}{-4}=0 \Rightarrow f(x)=p$

$A(2;-1)\in f \Rightarrow f(2)=-1$

$p=-1 \Rightarrow f(x)=-1$

c) $f(x)=mx+\dfrac{5}{2}$

$P(3;1)\in f \Rightarrow f(3)=1$

$m\cdot 3+\dfrac{5}{2}=1$

$3m=1-\dfrac{5}{2}$

$3m=-\dfrac{3}{2} \Rightarrow m=-\dfrac{1}{2} \Rightarrow f(x)=-\dfrac{1}{2}x+\dfrac{5}{2}$

d) $f(x) = mx + p$

$m = \frac{3-3}{-17-2} = \frac{0}{-19} = 0 \Rightarrow f(x) = p$

$k(2;3) \in f \Rightarrow f(2) = 3$

$p = 3 \Rightarrow f(x) = 3$

e) $f(x) = 2x - 1$

f) $f(x) = 0x + 2 = 2$

g) Coupe l'axe des x en -1 $\Rightarrow$ -1 est un zéro de la fct ou $B(-1;0) \in f$

$m = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1 \Rightarrow f(x) = 1x + p = x + p$

$A(2;3) \in f \Rightarrow f(2) = 3$

$2 + p = 3 \Rightarrow p = 1 \Rightarrow f(x) = x + 1$

Exercice 7

a) $f(x) = x^2 + 12x + 11 = (x+11)(x+1)$

$x_s = \frac{-b}{2a} = \frac{-12}{2} = -6$ $\quad y_s = f(-6) = (-6)^2 + 12\cdot(-6) + 11 = 36 - 72 + 11 = -36 + 11 = -25$

$S(-6;-25)$ est un minimum, car $a > 0 \Rightarrow f$ est convexe $\cup$.

b) $f(x) = x^2 + 4x = x(x+4)$

$x_s = \frac{-b}{2a} = \frac{-4}{2} = -2$ $\quad y_s = f(-2) = (-2)^2 + 4\cdot(-2) = 4 - 8 = -4$ $\quad S(-2;-4)$ minimum

c) $f(x) = x^2 - 2x - 3 = (x-3)(x+1)$

$x_s = \frac{2}{2} = 1$ $\quad y_s = f(1) = 1 - 2 - 3 = -4$ $\quad S(1;-4)$ minimum

d) $f(x) = -x^2 + 2x - 1 = -(x^2 - 2x + 1) = -(x-1)^2$

$x_s = \frac{-2}{-2} = 1$ $\quad y_s = f(1) = -1 + 2 - 1 = 0$ $\quad S(1;0)$ maximum

e) $f(x) = x^2 + x + 1$

$\Delta = b^2 - 4ac = 1^2 - 4\cdot1\cdot1 = 1 - 4 = -3 < 0 \Rightarrow$ pas de zéros $\Rightarrow$ pas de factorisation

$x_s = \frac{-1}{2}$ $\quad y_s = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{1}{4} - \frac{2}{4} + \frac{4}{4} = \frac{3}{4}$

$S\left(-\frac{1}{2};\frac{3}{4}\right)$ minimum

f) $f(x) = -\frac{1}{3}x^2 - \frac{1}{3}x - \frac{1}{3} = -\frac{1}{3}(x^2 + x + 1)$ $\Rightarrow$ pas factorisable cf e)

$x_S = \frac{\frac{1}{3}}{2\cdot(-\frac{1}{3})} = \frac{\frac{1}{3}}{-\frac{2}{3}} = \frac{1}{3}\cdot\left(-\frac{3}{2}\right) = -\frac{1}{2}$ $\quad y_S = f\left(-\frac{1}{2}\right) = -\frac{1}{3}\left(\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1\right) = -\frac{1}{3}\cdot\frac{3}{4} = -\frac{1}{4}$

$S\left(-\frac{1}{2}; -\frac{1}{4}\right)$ maximum

g) $f(x) = 2x^2 - x + 2$

$\Delta = (-1)^2 - 4\cdot 2\cdot 2 = 1 - 16 = -16 < 0$ $\Rightarrow$ pas factorisable

$x_S = \frac{1}{2\cdot 2} = \frac{1}{4}$ $\quad y_S = f\left(\frac{1}{4}\right) = 2\cdot\left(\frac{1}{4}\right)^2 - \frac{1}{4} + 2 = 2\cdot\frac{1}{16} - \frac{1}{4} + 2 = \frac{1}{8} - \frac{2}{8} + \frac{16}{8} = \frac{15}{8}$

$S\left(\frac{1}{4}; \frac{15}{8}\right)$ minimum

h) $f(x) = 3x^2 - 12x + 12 = 3(x^2 - 4x + 4) = 3(x-2)^2$

$S(2; 0)$ minimum

i) $f(x) = -x^2 + \frac{4}{3}x - \frac{4}{9} = -\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) = -\left(x - \frac{2}{3}\right)^2$

$S\left(\frac{2}{3}; 0\right)$ maximum

j) $f(x) = \frac{1}{2}x^2 + x + 1$

$\Delta = 1^2 - 4\cdot\frac{1}{2}\cdot 1 = 1 - 2 = -1 < 0$ $\Rightarrow$ pas factorisable

$x_S = \frac{-1}{2\cdot\frac{1}{2}} = -1$ $\quad y_S = f(x_S) = \frac{1}{2}(-1)^2 + (-1) + 1 = \frac{1}{2}$

$S\left(-1; \frac{1}{2}\right)$ minimum

k) $f(x) = -2x^2 - 5x + 3 = -2\left(x^2 + \frac{5}{2}x - \frac{3}{2}\right)$

$\Delta = (-5)^2 - 4\cdot(-2)\cdot 3 = 25 + 24 = 49 \Rightarrow \sqrt{\Delta} = \sqrt{49} = 7$

$x_{1,2} = \frac{5 \pm 7}{2\cdot(-2)} = \frac{5 \pm 7}{-4}$ $\quad x_1 = -3$ $\quad x_2 = \frac{2}{4} = \frac{1}{2}$

$f(x) = -2(x+3)\left(x - \frac{1}{2}\right)$

$x_S = \frac{5}{2\cdot(-2)} = -\frac{5}{4}$ $\quad y_S = f(x_S) = -2\cdot\left(-\frac{5}{4}\right)^2 - 5\cdot\left(-\frac{5}{4}\right) + 3 = -\overset{1}{\cancel{2}}\cdot\frac{25}{\underset{8}{\cancel{16}}} + \frac{25}{4} + 3 = -\frac{25}{8} + \frac{50}{8} + \frac{24}{8} = \frac{49}{8}$

$S\left(-\frac{5}{4}; \frac{49}{8}\right)$ maximum

l) $f(x) = 2x^2 - x + 4$

$\Delta = (-1)^2 - 4\cdot 2\cdot 4 = 1 - 32 = -31 < 0 \Rightarrow$ pas factorisable

$x_s = \frac{1}{2\cdot 2} = \frac{1}{4}$ $\quad y_s = f(x_s) = 2\cdot\left(\frac{1}{4}\right)^2 - \frac{1}{4} + 4 = \cancel{2}^{1}\cdot\frac{1}{\cancel{16}_{8}} - \frac{1}{4} + 4 = \frac{1}{8} - \frac{2}{8} + \frac{32}{8} = \frac{31}{8}$

$S\left(\frac{1}{4}; \frac{31}{8}\right)$ minimum

m) $f(x) = 7x^2 + 8x + 1$

$\Delta = 8^2 - 4\cdot 7\cdot 1 = 64 - 28 = 36$ $\quad \sqrt{\Delta} = \sqrt{36} = 6$

$x_{1,2} = \frac{-8 \pm 6}{2\cdot 7} = \frac{-8 \pm 6}{14}$ $\quad x_1 = -\frac{1}{7}$ $\quad x_2 = -1$

$f(x) = 7\left(x + \frac{1}{7}\right)(x+1)$

$x_s = \frac{-8}{2\cdot 7} = -\frac{8}{14} = -\frac{4}{7}$ $\quad y_s = f(x_s) = 7\cdot\left(-\frac{4}{7}\right)^2 + 8\cdot\left(-\frac{4}{7}\right) + 1 = \cancel{7}^{1}\cdot\frac{16}{\cancel{49}_{7}} - \frac{32}{7} + \frac{7}{7} = -\frac{9}{7}$

$S\left(-\frac{4}{7}; -\frac{9}{7}\right)$ minimum

n) $f(x) = 15x^2 + x - 2$

$\Delta = 1^2 - 4\cdot 15\cdot(-2) = 1 + 120 = 121$ $\quad \sqrt{121} = 11$

$x_{1,2} = \frac{-1 \pm 11}{2\cdot 15} = \frac{-1 \pm 11}{30}$ $\quad x_1 = -\frac{12}{30} = -\frac{2}{5}$ $\quad x_2 = \frac{10}{30} = \frac{1}{3}$

$f(x) = 15\left(x + \frac{2}{5}\right)\left(x - \frac{1}{3}\right)$

$x_s = \frac{-1}{2\cdot 15} = -\frac{1}{30}$ $\quad y_s = f(x_s) = 15\cdot\left(-\frac{1}{30}\right)^2 + \left(-\frac{1}{30}\right) - 2 = \cancel{15}^{1}\cdot\frac{1}{\cancel{900}_{60}} - \frac{1}{30} - 2 = \frac{1}{60} - \frac{2}{60} - \frac{120}{60} = -\frac{121}{60}$

$S\left(-\frac{1}{30}; -\frac{121}{60}\right)$ minimum

o) $f(x) = -x^2 + 8x + 20 = -(x^2 - 8x - 20) = -(x-10)(x+2)$

$x_s = \frac{-8}{2\cdot(-1)} = 4$ $\quad y_s = f(x_s) = -4^2 + 8\cdot 4 + 20 = -16 + 32 + 20 = 36$

$S(4; 36)$ maximum

p) $f(x) = x^2 - 5 = (x-\sqrt{5})(x+\sqrt{5})$

$x_s = 0 \quad y_s = f(x_s) = 0^2 - 5 = -5$

$S(0; -5)$ minimum

q) $f(x) = x^2 + 4x - 6$

$\Delta = 4^2 - 4 \cdot 1 \cdot (-6) = 16 + 24 = 40 \qquad \sqrt{\Delta} = \sqrt{40} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

$x_{1,2} = \frac{-4 \pm 2\sqrt{10}}{2} = \frac{\cancel{2}(-2 \pm \sqrt{10})}{\cancel{2}} = \begin{cases} x_1 = -2 + \sqrt{10} \\ x_2 = -2 - \sqrt{10} \end{cases}$

$f(x) = (x - (-2+\sqrt{10}))(x - (-2-\sqrt{10})) = (x + 2 - \sqrt{10})(x + 2 + \sqrt{10})$

$x_s = \frac{-4}{2 \cdot 1} = -2 \qquad y_s = f(x_s) = (-2)^2 + 4 \cdot (-2) - 6 = 4 - 8 - 6 = -10$

$S(-2; -10)$ minimum

r) $f(x) = 3x^2 + 5x - 4$

$\Delta = 5^2 - 4 \cdot 3 \cdot (-4) = 25 + 48 = 73$

$x_{1,2} = \frac{-5 \pm \sqrt{73}}{2 \cdot 3} = \frac{-5 \pm \sqrt{73}}{6} \begin{cases} x_1 = \frac{-5 + \sqrt{73}}{6} \\ x_2 = \frac{-5 - \sqrt{73}}{6} \end{cases}$

$f(x) = \left(x - \left(\frac{-5+\sqrt{73}}{6}\right)\right)\left(x - \left(\frac{-5-\sqrt{73}}{6}\right)\right) = \left(x - \frac{-5+\sqrt{73}}{6}\right)\left(x - \frac{-5-\sqrt{73}}{6}\right)$

$x_s = \frac{-5}{2 \cdot 3} = -\frac{5}{6} \qquad y_s = f(x_s) = 3 \cdot \left(-\frac{5}{6}\right)^2 + 5 \cdot \left(-\frac{5}{6}\right) - 4 = \frac{\cancel{3}^{1} \cdot 25}{\cancel{36}_{12}} - \frac{25}{6} - 4 = \frac{25}{12} - \frac{50}{12} - \frac{48}{12} = -\frac{73}{12}$

$S\left(-\frac{5}{6}; -\frac{73}{12}\right)$ minimum

Exercice 9

$f(x) = ax^2 + bx + c$

$A(2;9) \in f \Rightarrow f(2)=9 \Rightarrow \begin{cases} a\cdot 2^2 + b\cdot 2 + c = 9 \\ a\cdot(-6)^2 + b\cdot(-6) + c = -7 \\ a\cdot 1^2 + b\cdot 1 + c = 0 \end{cases} \Rightarrow \begin{cases} 4a + 2b + c = 9 & (1) \\ 36a - 6b + c = -7 & (2) \\ a + b + c = 0 & (3) \end{cases}$

$B(-6;-7) \in f \Rightarrow f(-6) = -7$

$C(1;0) \in f \Rightarrow f(1) = 0$

de (3) on obtient que $c = -a - b$

$\begin{cases} 4a + 2b - a - b = 9 & (1) \\ 36a - 6b - a - b = -7 & (2) \end{cases}$ $\quad \begin{cases} 3a + b = 9 \\ 35a - 7b = -7 \quad :7 \end{cases}$ $\quad + \begin{cases} 3a + b = 9 \\ 5a - b = -1 \end{cases}$

$8a = 8 \Rightarrow \underline{\underline{a=1}}$

a ds (1) : $3 + b = 9 \Rightarrow \underline{\underline{b = 6}}$

$c = -1 - 6 = -7$

$f(x) = x^2 + 6x - 7$

Exercice 10

a) $(3x+1)(2x+3) > 0$

$x = -\frac{1}{3}$ $\quad$ $x = -\frac{3}{2}$

x	$-\infty$		$-\frac{3}{2}$		$-\frac{1}{3}$		$+\infty$
$3x+1$		$-$		$-$	0	$+$	
$2x+3$		$-$	0	$+$		$+$	
$(3x+1)(2x+3)$		$+$	0	$-$	0	$+$	

$S =]-\infty; -\frac{3}{2}[\cup]-\frac{1}{3}; +\infty[$

b) $(5-x)(2x+1) < 0$

$x = 5$ $\quad$ $x = -\frac{1}{2}$

x	$-\infty$		$-\frac{1}{2}$		5		$+\infty$
$5-x$		$+$		$+$	0	$-$	
$2x+1$		$-$	0	$+$		$+$	
$(5-x)(2x+1)$		$-$	0	$+$	0	$-$	

$S =]-\infty; -\frac{1}{2}[\cup]5; +\infty[$

c) $\frac{5+2x}{4x+1} \leq 0$

$x = -\frac{5}{2}$

$x \neq -\frac{1}{4}$

x	$-\infty$		$-\frac{5}{2}$		$-\frac{1}{4}$		$+\infty$
$5+2x$		$-$	0	$+$		$+$	
$4x+1$		$-$		$-$	0	$+$	
$\frac{5+2x}{4x+1}$		$+$	0	$-$		$+$	

$S = [-\frac{5}{2}; -\frac{1}{4}[$

d) $\frac{2x+1}{2-x} \geq 0$

$x = -\frac{1}{2}$

$x \neq 2$

x	$-\infty$		$-\frac{1}{2}$		2		$+\infty$
$2x+1$		$-$	0	$+$		$+$	
$2-x$		$+$		$+$	0	$-$	
$\frac{2x+1}{2-x}$		$-$	0	$+$		$-$	

$S = [-\frac{1}{2}; 2[$