Fonctions polynomiales - Solutions détaillées

Exercice 12

a) $f(x) = \frac{2}{3}x^2 + \frac{1}{2}x = x\left(\frac{2}{3}x + \frac{1}{2}\right)$

$x\left(\frac{2}{3}x + \frac{1}{2}\right) = 0$

$x = 0$ ou $\frac{2}{3}x + \frac{1}{2} = 0 \quad | \cdot 6$

$4x + 3 = 0 \quad | -3$

$4x = -3 \quad | :4$

$x = -\frac{3}{4}$

$x_1 = 0$ et $x_2 = -\frac{3}{4}$

b) $f(x) = (1-2x)(x-2)$

$(1-2x)(x-2) = 0$

$1-2x = 0$ ou $x-2=0$

$1 = 2x$; $x = 2$

$\frac{1}{2} = x$

$x_1 = \frac{1}{2}$ et $x_2 = 2$

c) $f(x) = \left(\frac{2}{5}x - \frac{6}{5}\right)(x^2+4)$

$\left(\frac{2}{5}x - \frac{6}{5}\right)(x^2+4) = 0$

$\frac{2}{5}x - \frac{6}{5} = 0$ ou $x^2 + 4 = 0$

$2x - 6 = 0$; pas de solution

$2x = 6$

$x = 3$

$x_1 = 3$

d) $f(x) = x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x-3)(x+2)$

$x(x-3)(x+2) = 0$

$x = 0$ ou $x - 3 = 0$ ou $x + 2 = 0$

$x = 3$; $x = -2$

$x_1 = 0$, $x_2 = 3$ et $x_3 = -2$

Exercice 14

a)

$$
\begin{array}{l|l}
x^3 + 2x^2 - 17x + 6 & x-3 \\
-(x^3 - 3x^2) & \hline \\
\quad 5x^2 - 17x & x^2 + 5x - 2 \\
\quad -(5x^2 - 15x) & \\
\qquad -2x + 6 & \\
\qquad -(2x + 6) & \\
\qquad\quad 0 &
\end{array}
$$

$f(x) = (x-3)(x^2+5x-2)$

b)

$$
\begin{array}{l|l}
2x^3 + 2x^2 - 21x + 12 & x+4 \\
-(2x^3 + 8x^2) & \hline \\
\quad -6x^2 - 21x & 2x^2 - 6x + 3 \\
\quad -(-6x^2 - 24x) & \\
\qquad 3x + 12 & \\
\qquad -(3x + 12) & \\
\qquad\quad 0 &
\end{array}
$$

$f(x) = (x+4)(2x^2-6x+3)$

c) $2x^3 - 7x^2 - x + 2$

$-(2x^3 - x^2)$

$-6x^2 - x$

$-(-6x^2 + 3x)$

$-4x + 2$

$-(-4x + 2)$

0

Divisor: $2x - 1$; quotient: $x^2 - 3x - 2$

$$f(x) = (2x-1)(x^2-3x-2)$$

d) $x^4 + 2x^3 - 4x^2 - 9x - 2$

$-(x^4 + 2x^3)$

$0 - 4x^2 - 9x$

$-(-4x^2 - 8x)$

$-x - 2$

$-(-x-2)$

0

Divisor: $x + 2$; quotient: $x^3 - 4x - 1$

$$f(x) = (x+2)(x^3 - 4x - 1)$$

Exercice 15

a) $x^3 - 8x^2 + 16x + 4$

$-(x^3 - 5x^2)$

$-3x^2 + 16x$

$-(-3x^2 + 15x)$

$x + 4$

$-(x - 5)$

9

Divisor: $x - 5$; quotient: $x^2 - 3x + 1$

$$f(x) = (x-5)(x^2-3x+1) + 9$$

b) $x^5 + 0x^4 - 8x^3 + 8x^2 - 7x + 6$

$-(x^5 + 3x^4 - 2x^3)$

$-3x^4 - 6x^3 + 8x^2$

$-(-3x^4 - 9x^3 + 6x^2)$

$3x^3 + 2x^2 - 7x$

$-(3x^3 + 9x^2 - 6x)$

$-7x^2 - x + 6$

$-(-7x^2 - 21x + 14)$

$20x - 8$

Divisor: $x^2 + 3x - 2$; quotient: $x^3 - 3x^2 + 3x - 7$

$$f(x) = (x^2+3x-2)(x^3-3x^2+3x-7) + 20x - 8$$

c) $5x^3 - 2x^2 + 4x - 4$

$-(5x^3 - 5x^2)$

$3x^2 + 4x$

$-(3x^2 - 3x)$

$7x - 4$

$-(7x - 7)$

3

Divisor: $x - 1$; quotient: $5x^2 + 3x + 7$

$$f(x) = (x-1)(5x^2+3x+7) + 3$$

d) $6x^5 - x^4 - 35x^3 + 31x^2 - 10x + 6$

$-(6x^5 - 15x^4 + 6x^3)$

$14x^4 - 41x^3 + 31x^2$

$-(14x^4 - 35x^3 + 14x^2)$

$-6x^3 + 17x^2 - 10x$

$-(-6x^3 + 15x^2 - 6x)$

$2x^2 - 4x + 6$

$-(2x^2 - 5x + 2)$

$x + 4$

Divisor: $2x^2 - 5x + 2$; quotient: $3x^3 + 7x^2 - 3x + 1$

$$f(x) = (2x^2-5x+2)(3x^3+7x^2-3x+1) + x + 4$$

e) $3x^4 - 23x^3 + 26x^2 + 28x - 24 \quad | \quad x^2 - 5x - 6$

$-(3x^4 - 15x^3 - 18x^2) \qquad\qquad | \quad 3x^2 - 8x + 4$

$-8x^3 + 44x^2 + 28x$

$-(-8x^3 + 40x^2 + 48x)$

$4x^2 - 20x - 24$

$-(4x^2 - 20x - 24)$

0

$$f(x) = (x^2 - 5x - 6)(3x^2 - 8x + 4)$$

f) $-x^3 - x^2 + 0x + 5 \quad | \quad 2x - 3$

$-(-x^3 + \frac{3}{2}x^2) \qquad\qquad | \quad -\frac{1}{2}x^2 - \frac{5}{4}x - \frac{15}{8}$

$-\frac{5}{2}x^2 + 0x$

$-(-\frac{5}{2}x^2 + \frac{15}{4}x)$

$-\frac{15}{4}x + 5$

$-(-\frac{15}{4}x + \frac{45}{8})$

$-\frac{5}{8}$

$$f(x) = (2x-3)\left(-\frac{1}{2}x^2 - \frac{5}{4}x - \frac{15}{8}\right) - \frac{5}{8}$$

g) $x^4 + x^3 - 8x^2 - 3x + 19 \quad | \quad -x^2 + x + 2$

$-(x^4 - x^3 - 2x^2) \qquad\qquad | \quad -x^2 - 2x + 4$

$2x^3 - 6x^2 - 3x$

$-(2x^3 - 2x^2 - 4x)$

$-4x^2 + x + 19$

$-(-4x^2 + 4x + 8)$

$-3x + 11$

$$f(x) = (-x^2 + x + 2)(-x^2 - 2x + 4) - 3x + 11$$

Exercice 16

a) $f(-x) = -6 \cdot (-x)^2 + 1 = -6x^2 + 1 = f(x) \quad \Rightarrow \quad f$ est paire

b) $f(-x) = 3 \cdot (-x) = -3x = -f(x) \quad \Rightarrow \quad f$ est impaire

c) $f(-x) = (-x)^3 - 2 \cdot (-x)^2 + (-x) = -x^3 - 2x^2 - x \quad \Rightarrow \quad f$ est ni paire, ni impaire

d) $f(-x) = (-x)^9 - (-x)^7 + (-x)^5 - (-x)^3 = -x^9 + x^7 - x^5 + x^3 = -(x^9 - x^7 + x^5 - x^3) = -f(x)$

$\Rightarrow \quad f$ est impaire

e) $f(-x) = 4 = f(x) \quad \Rightarrow \quad f$ est paire

f) $f(-x) = (-x) - 4 = -x - 4 \quad \Rightarrow \quad f$ est ni paire, ni impaire

g) $f(-x) = 4 \cdot (-x)^4 + (-x)^2 = 4x^4 + x^2 = f(x) \quad \Rightarrow \quad f$ est paire

h) $f(-x) = (-x)^{100} = x^{100} = f(x) \quad \Rightarrow \quad f$ est paire

Exercice 20

a) substituons x^2 par y: $y^2 - 13y + 36 = 0$ factorisation

$(y-9)(y-4) = 0$

$y - 9 = 0$ ou $y - 4 = 0$

remplaçons y par x^2: $x^2 - 9 = 0$ $x^2 - 4 = 0$

$(x-3)(x+3) = 0$ $(x-2)(x+2) = 0$

$x_1 = 3$ $x_2 = -3$ $x_3 = 2$ $x_4 = -2$

b) Substituons x^2 par y: $16y^2 - 40y + 9 = 0$

$\Delta = (-40)^2 - 4 \cdot 16 \cdot 9 = 1024$ $\sqrt{1024} = 32$

$y_{1,2} = \dfrac{40 \pm 32}{2 \cdot 16} = \dfrac{40 \pm 32}{32}$ $\rightarrow y_1 = \dfrac{72}{32} = \dfrac{9}{4}$ $\rightarrow y_2 = \dfrac{8}{32} = \dfrac{1}{4}$

$x^2 = \dfrac{9}{4}$ ou $x^2 = \dfrac{1}{4}$

$x^2 - \dfrac{9}{4} = 0$ $x^2 - \dfrac{1}{4} = 0$

$\left(x - \dfrac{3}{4}\right)\left(x + \dfrac{3}{4}\right) = 0$ $\left(x - \dfrac{1}{2}\right)\left(x + \dfrac{1}{2}\right) = 0$

$x_1 = \dfrac{3}{4}$ $x_2 = -\dfrac{3}{4}$ $x_3 = \dfrac{1}{2}$ $x_4 = -\dfrac{1}{2}$

c) Substituons x^3 par y: $y^2 - 19y - 216 = 0$

$\Delta = (-19)^2 - 4 \cdot 1 \cdot (-216) = 1225$ $\sqrt{1225} = 35$

$y_{1,2} = \dfrac{19 \pm 35}{2}$ $\rightarrow y_1 = \dfrac{54}{2} = 27$ $\rightarrow y_2 = -\dfrac{16}{2} = -8$

$x^3 = 27$ ou $x^3 = -8$

$x_1 = \sqrt[3]{27} = 3$ $x_2 = \sqrt[3]{-8} = -2$

Exercice 21

a) $f(x) = 3x^3 + 2x^2 - 7x + 2$

Coefficient dominant $\neq 1$ $\Rightarrow$ on essaie avec 1.

$f(1) = 3 + 2 - 7 + 2 = 7 - 7 = 0$ $\Rightarrow$ 1 est un zéro de f $\Rightarrow$ $f(x) = (x-1) \cdot g(x)$

On sait que 1 est un zéro :

$$\begin{array}{l|l} 3x^3+2x^2-7x+2 & x-1 \\ \cline{2-2} -(3x^3-3x^2) & 3x^2+5x-2 \\ \quad 5x^2-7x & \\ \quad -(5x^2-5x) & \\ \qquad -2x+2 & \\ \qquad -(-2x+2) & \\ \qquad\quad 0 & \end{array}$$

$3x^2+5x-2 = 3(x+2)(x-\frac{1}{3})$

$\Delta = 5^2 - 4\cdot 3\cdot(-2) = 25+24 = 49$

$x_{1,2} = \frac{-5 \pm \sqrt{49}}{2\cdot 3} = \frac{-5\pm 7}{6}$; $x_1 = -2$; $x_2 = \frac{1}{3}$

$f(x) = (x-1)(3x^2+5x-2) = (x-1)(x+2)(x-\frac{1}{3})\cdot 3 = (x-1)(x+2)\cdot(3x-1)$

$3x^2 + 2x^2 - 7x + 2 = (x-1)(x+2)(3x-1)$

b) $f(x) = x^3 - 3x^2 + 3x - 2$

Diviseurs entiers de -2 : $-1 ; 2 ; 1 ; -2$

$f(-1) = -1-3-3-2 = -9$ $\quad f(2) = 8-12+6-2 = 0$ $\Rightarrow$ 2 est un zéro de f.

$f(1) = 1-3+3-2 = -1$ $\quad f(-2) = -8-12-6-2 = -28$

$$\begin{array}{l|l} x^3-3x^2+3x-2 & x-2 \\ \cline{2-2} -(x^3-2x^2) & x^2-x+1 \\ \quad -x^2+3x & \\ \quad -(-x^2+2x) & \\ \qquad x-2 & \end{array}$$

pas factorisable

$\Delta = (-1)^2 - 4\cdot 1\cdot 1 = -3 < 0$

$f(x) = (x-2)(x^2-x+1)$

c) $x^3 + 2x^2 - 5x - 6$

Diviseurs entiers de -6 : $1 ; -6 ; -1 ; 6 ; 2 ; -3 ; -2 ; 3$

$f(1) = -8$ $\quad f(-6) = -120$ $\quad f(-1) = 0$ $\quad f(6) = 252$ $\quad f(2) = 0$ $\quad f(-3) = 0$

$f(-2) = 4$ $\quad f(3) = 24$ $\quad \Rightarrow -1, 2$ et -3 sont des zéros de f.

f est de degré 3, elle a au plus 3 zéros :

$f(x) = (x+1)(x-2)(x+3)$

d) $x^4 - 7x^3 + 17x^2 - 17x + 6$

Diviseurs entiers de $+6$: $1 ; 6 ; -1 ; -6 ; 2 ; 3 ; -2 ; -3$

$f(1) = 0$ $\quad f(6) = 300$ $\quad f(-1) = 48$ $\quad f(-6) = 3528$ $\quad f(2) = 0$ $\quad f(3) = 0$

$f(-2) = 180$ $\quad f(-3) = 480$ $\quad \Rightarrow$ 1, 2 et 3 sont des zéros de f.

$$\begin{array}{l|l} x^4-7x^3+17x^2-17x+6 & x-1 \\ \cline{2-2} -(x^4-x^3) & x^3-6x^2+11x-6 \\ \quad -6x^3+17x^2 & \\ \quad -(-6x^3+6x^2) & \\ \qquad 11x^2-17x & \\ \qquad -(11x^2-11x) & \\ \qquad\quad -6x+6 & \\ \qquad\quad -(-6x+6) & \\ \qquad\qquad 0 & \end{array}$$

(1 est zéro de f)

$f(x) = (x-1)(x^3-6x^2+11x-6)$

$$\begin{array}{l} x^3 - 6x^2 + 11x - 6 \\ -(x^3 - 2x^2) \\ \quad -4x^2 + 11x \\ \quad -(-4x^2 + 8x) \\ \qquad 3x - 6 \\ \qquad -(3x-6) \\ \qquad\quad 0 \end{array} \quad \begin{array}{|l} x-2 \\ \hline x^2 - 4x + 3 \end{array}$$

(2 est zéro de f)

$f(x) = (x-1)(x-2)(x^2-4x+3)$

$$\begin{array}{l} x^2 - 4x + 3 \\ -(x^2 - 3x) \\ \quad -x + 3 \\ \quad -(-x+3) \\ \qquad 0 \end{array} \quad \begin{array}{|l} x-3 \\ \hline x - 1 \end{array}$$

(3 est zéro de f)

$f(x) = (x-1)(x-2)(x-3)(x-1) = (x-1)^2(x-2)(x-3)$

e) $f(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$

Diviseurs entiers de +15 : 3 ; 5 ; -3 ; -5 ; 1 ; 15 ; -1 ; -15

$f(3) = 0$ $\quad f(5) = 480$ $\quad f(-3) = -96$ $\quad f(-5) = 0$ $\quad f(1) = 0$ $\quad f(15) = 53760$

$f(-1) = 0$ $\quad f(-15) = 40320$ $\quad \Rightarrow$ 3, -5, 1 et -1 sont des zéro de f.

f est de degré 4, elle a donc au plus 4 zéros.

$f(x) = (x-3)(x+5)(x-1)(x+1)$

f) $f(x) = x^5 + 3x^4 - 16x - 48 = x^4(x+3) - 16(x+3) = (x^4-16)(x+3)$

$= (x^2+4)(x^2-4)(x+3) = (x^2+4)(x+2)(x-2)(x+3)$

g) $f(x) = 6x^4 + 13x^3 - 13x - 6$

Coefficient dominant $\neq 1$ $\Rightarrow$ on essaie avec 1, -1, 2, ..

$f(-1) = 0$ $\quad f(1) = 0$ $\quad \Rightarrow$ -1 et 1 sont des zéros de f.

$$\begin{array}{l} 6x^4 + 13x^3 + 0x^2 - 13x - 6 \\ -(6x^4 + 6x^3) \\ \quad 7x^3 + 0x^2 \\ \quad -(7x^3 + 7x^2) \\ \qquad -7x^2 - 13x \\ \qquad -(-7x^2 - 7x) \\ \qquad\quad -6x - 6 \\ \qquad\quad -(-6x-6) \\ \qquad\qquad 0 \end{array} \quad \begin{array}{|l} x+1 \\ \hline 6x^3 + 7x^2 - 7x - 6 \end{array}$$

(-1 est zéro de f)

$f(x) = (x+1)(6x^3 + 7x^2 - 7x - 6)$

$$\begin{array}{l} 6x^3 + 7x^2 - 7x - 6 \\ -(6x^3 - 6x^2) \\ \quad 13x^2 - 7x \\ \quad -(13x^2 - 13x) \\ \qquad 6x - 6 \\ \qquad -(6x-6) \\ \qquad\quad 0 \end{array} \quad \begin{array}{|l} x-1 \\ \hline 6x^2 + 13x + 6 \end{array}$$

(1 est zéro de f)

$f(x) = (x+1)(x-1)(6x^2 + 13x + 6)$

factorisation de $6x^2+13x+6$:

$\Delta = 13^2 - 4\cdot6\cdot6 = 25$ $\quad \sqrt{\Delta} = \sqrt{25} = 5$

$x_{1,2} = \dfrac{-13 \pm 5}{2\cdot 6} = \dfrac{-13 \pm 5}{12}$ $\quad x_1 = \dfrac{-8}{12} = -\dfrac{2}{3}$ $\quad x_2 = \dfrac{-18}{12} = -\dfrac{3}{2}$

$6x^2+13x+6 = 6\left(x+\frac{2}{3}\right)\left(x+\frac{3}{2}\right)$

$f(x) = 6(x+1)(x-1)\left(x+\frac{2}{3}\right)\left(x+\frac{3}{2}\right) = (x+1)(x-1)\,3\left(x+\frac{2}{3}\right)2\left(x+\frac{3}{2}\right) = (x+1)(x-1)(3x+2)(2x+3)$

h) $f(x) = x^4 - 3x^3 + 3x^2 - 3x + 2$

Diviseurs entiers de $+2$: $1 ; 2 ; -1 ; -2$

$f(1) = 0$ $\quad f(2) = 0$ $\quad f(-1) = 12$ $\quad f(-2) = 60$ $\Rightarrow$ 1 et 2 sont des zéros de f.

$$\begin{array}{l|l}
x^4 - 3x^3 + 3x^2 - 3x + 2 & x-1 \\
-(x^4 - x^3) & \overline{x^3 - 2x^2 + x - 2} \\
\quad -2x^3 + 3x^2 & \\
\quad -(-2x^3 + 2x^2) & \\
\qquad x^2 - 3x & \\
\qquad -(x^2 - x) & \\
\qquad\quad -2x + 2 & \\
\qquad\quad -(-2x+2) & \\
\qquad\qquad 0 &
\end{array}$$

(1 est un zéro de f)

$f(x) = (x-1)(x^3 - 2x^2 + x - 2)$

$$\begin{array}{l|l}
x^3 - 2x^2 + x - 2 & x-2 \\
-(x^3 - 2x^2) & \overline{x^2 + 1} \\
\quad 0 + x - 2 & \\
\quad -(x-2) & \\
\qquad 0 &
\end{array}$$

(2 est un zéro de f)

pas factorisable

$f(x) = (x-1)(x-2)(x^2+1)$

i) $f(x) = 12x^3 - 7x^2 - 8x + 3$

Coefficient dominant $\neq 1$ $\Rightarrow$ on essaie avec $1, -1, 2, \ldots$

$f(1) = 0 \Rightarrow 1$ est un zéro de f

$$\begin{array}{l|l}
12x^3 - 7x^2 - 8x + 3 & x-1 \\
-(12x^3 - 12x^2) & \overline{12x^2 + 5x - 3} \\
\quad 5x^2 - 8x & \\
\quad -(5x^2 - 5x) & \\
\qquad -3x + 3 & \\
\qquad -(-3x + 3) & \\
\qquad\quad 0 &
\end{array}$$

(1 est un zéro de f)

$f(x) = (x-1)(12x^2 + 5x - 3)$

factorisons $12x^2 + 5x - 3$

$\Delta = 5^2 - 4\cdot12\cdot(-3) = 169$ $\quad \sqrt{\Delta} = \sqrt{169} = 13$

$x_{1,2} = \dfrac{-5 \pm 13}{2\cdot12} = \dfrac{-5 \pm 13}{24}$ $\quad x_1 = \dfrac{8}{24} = \dfrac{1}{3}$ $\quad x_2 = \dfrac{-18}{24} = -\dfrac{3}{4}$

$12x^2 + 5x - 3 = 12\left(x - \frac{1}{3}\right)\left(x + \frac{3}{4}\right)$

$f(x) = 12(x-1)\left(x-\frac{1}{3}\right)\left(x+\frac{3}{4}\right) = (x-1)\,3\left(x-\frac{1}{3}\right)4\left(x+\frac{3}{4}\right) = (x-1)(3x-1)(4x+3)$

j) $6x^4 - 5x^3 - 23x^2 + 20x - 4$

Coefficient dominant $\neq 1$ $\Rightarrow$ on essaie avec $1, -1, 2, -2, \ldots$

$f(1) = -6$ $\quad f(-1) = -36$ $\quad f(2) = 0$ $\quad f(-2) = 0$

$\Rightarrow$ 2 et -2 sont des zéros de f.

$$\begin{array}{l|l}
6x^4 - 5x^3 - 23x^2 + 20x - 4 & x-2 \\
-(6x^4 - 12x^3) & \overline{6x^3 + 7x^2 - 9x + 2} \\
\quad 7x^3 - 23x^2 & \\
\quad -(7x^3 - 14x^2) & \\
\quad\quad -9x^2 + 20x & \\
\quad\quad -(-9x^2 + 18x) & \\
\quad\quad\quad 2x - 4 & \\
\quad\quad\quad -(2x - 4) & \\
\quad\quad\quad\quad 0 &
\end{array}$$

(2 est un zéro de f)

$f(x) = (x-2)(6x^3 + 7x^2 - 9x + 2)$

$$\begin{array}{l|l}
6x^3 + 7x^2 - 9x + 2 & x+2 \\
-(6x^3 + 12x^2) & \overline{6x^2 - 5x + 1} \\
\quad -5x^2 - 9x & \\
\quad -(-5x^2 - 10x) & \\
\quad\quad x + 2 & \\
\quad\quad -(x+2) & \\
\quad\quad\quad 0 &
\end{array}$$

(-2 est zéro de f)

$f(x) = (x-2)(x+2)(6x^2 - 5x + 1)$

Factorisons $6x^2 - 5x + 1$

$\Delta = (-5)^2 - 4 \cdot 6 \cdot 1 = 25 - 24 = 1$ $\qquad \sqrt{\Delta} = \sqrt{1} = 1$

$x_{1,2} = \dfrac{5 \pm 1}{2 \cdot 6} = \dfrac{5 \pm 1}{12}$ $\quad x_1 = \dfrac{6}{12} = \dfrac{1}{2}$ $\quad x_2 = \dfrac{4}{12} = \dfrac{1}{3}$

$6x^2 - 5x + 1 = 6\left(x - \frac{1}{2}\right)\left(x - \frac{1}{3}\right)$

$f(x) = 6(x-2)(x+2)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right) = (x-2)(x+2)\, 2\left(x-\frac{1}{2}\right) 3\left(x-\frac{1}{3}\right) = (x-2)(x+2)(2x-1)(3x-1)$

k) $f(x) = 6x^4 + 4x^3 - 26x^2 - 16x + 8$

Coeff. dom $\neq 1$, on essaie avec $1, -1, 2, -2, \ldots$

$f(1) = -24$ $\quad f(-1) = 0$ $\quad f(2) = 0$ $\quad f(-2) = 0$

$\Rightarrow$ -1, 2 et -2 sont des zéros de f.

$$\begin{array}{l|l}
6x^4 + 4x^3 - 26x^2 - 16x + 8 & x+1 \\
-(6x^4 + 6x^3) & \overline{6x^3 - 2x^2 - 24x + 8} \\
\quad -2x^3 - 26x^2 & \\
\quad -(-2x^3 - 2x^2) & \\
\quad\quad -24x^2 - 16x & \\
\quad\quad -(-24x^2 - 24x) & \\
\quad\quad\quad 8x + 8 & \\
\quad\quad\quad -(8x + 8) & \\
\quad\quad\quad\quad 0 &
\end{array}$$

(-1 est un zéro de f)

$f(x) = (x+1)(6x^3 - 2x^2 - 24x + 8)$

$$\begin{array}{l} 6x^3-2x^2-24x+8 \\ -(6x^3-12x^2) \\ \quad 10x^2-24x \\ \quad -(10x^2-20x) \\ \qquad -4x+8 \\ \qquad -(-4x+8) \\ \qquad\quad 0 \end{array} \quad \begin{array}{|l} x-2 \\ \hline 6x^2+10x-4 \end{array}$$

(2 est un zéro de f)

$$f(x) = (x+1)(x-2)(6x^2+10x-4)$$

$$\begin{array}{l} 6x^2+10x-4 \\ -(6x^2+12x) \\ \quad -2x-4 \\ \quad -(-2x-4) \\ \qquad 0 \end{array} \quad \begin{array}{|l} x+2 \\ \hline 6x-2 \end{array}$$

(-2 est un zéro de f)

$$f(x) = (x+1)(x-2)(x+2)(6x-2) = 2(x+1)(x-2)(x+2)(3x-1)$$

e) $f(x) = 35x^4 - 57x^3 - 185x^2 + 129x - 18$

Coeff. dom $\neq 1$ $\Rightarrow$ on essaie avec $1, -1, 2, -2, \ldots$

$f(-1) = -240$ $\quad f(1) = -96$ $\quad f(2) = -396$ $\quad f(-2) = 0$ $\quad f(3) = 0$

$\Rightarrow$ -2 et 3 sont des zéros de f.

$$\begin{array}{l} 35x^4-57x^3-185x^2+129x-18 \\ -(35x^4+70x^3) \\ \quad -127x^3-185x^2 \\ \quad -(-127x^3-254x^2) \\ \qquad 69x^2+129x \\ \qquad -(69x^2+138x) \\ \qquad\quad -9x-18 \\ \qquad\quad -(-9x-18) \\ \qquad\qquad 0 \end{array} \quad \begin{array}{|l} x+2 \\ \hline 35x^3-127x^2+69x-9 \end{array}$$

(-2 est un zéro de f)

$$f(x) = (x+2)(35x^3-127x^2+69x-9)$$

$$\begin{array}{l} 35x^3-127x^2+69x-9 \\ -(35x^3-105x^2) \\ \quad -22x^2+69x \\ \quad -(-22x^2+66x) \\ \qquad 3x-9 \\ \qquad -(3x-9) \\ \qquad\quad 0 \end{array} \quad \begin{array}{|l} x-3 \\ \hline 35x^2-22x+3 \end{array}$$

(3 est un zéro de f)

$$f(x) = (x+2)(x-3)(35x^2-22x+3)$$

factorisons $35x^2-22x+3$

$\Delta = (-22)^2 - 4\cdot 35\cdot 3 = 64$ $\qquad \sqrt{\Delta} = \sqrt{64} = 8$

$$x_{1,2} = \frac{22 \pm 8}{2\cdot 35} = \frac{22\pm 8}{70} \qquad x_1 = \frac{30}{70} = \frac{3}{7} \qquad x_2 = \frac{14}{70} = \frac{1}{5}$$

$$35x^2-22x+3 = 35\left(x-\frac{3}{7}\right)\left(x-\frac{1}{5}\right)$$

$$f(x) = 35(x+2)(x-3)\left(x-\frac{3}{7}\right)\left(x-\frac{1}{5}\right) = (x+2)(x-3)\,7\left(x-\frac{3}{7}\right)5\left(x-\frac{1}{5}\right) = (x+2)(x-3)(7x-3)(5x-1)$$

Exercice 22

a) $f(x) = a(x-x_1)(x-x_2)(x-x_3) = a(x-4)(x-(-2))(x-5) = a(x-4)(x+2)(x-5)$

$f(1) = 72 \Rightarrow a(1-4)(1+2)(1-5) = 72$

$a \cdot (-3)(3)(-4) = 72$

$36a = 72$

$a = \frac{72}{36} = 2$

$f(x) = 2(x-4)(x+2)(x-5)$

b) $g(x) = a(x-0)(x-(-1))(x-2)(x-3) = ax(x+1)(x-2)(x-3)$

$(4; -20) \in g \Rightarrow g(4) = -20 \Rightarrow a \cdot 4(4+1)(4-2)(4-3) = -20$

$a \cdot 4 \cdot 5 \cdot 2 \cdot 1 = -20$

$40a = -20$

$a = \frac{-20}{40} = -\frac{1}{2}$

$g(x) = -\frac{1}{2}x(x+1)(x-2)(x-3)$

c) $h(x) = a(x-x_1)^2 = a(x-3)^2$

$(5; 2) \in h \Rightarrow h(5) = 2 \Rightarrow a(5-3)^2 = 2$

$a(2)^2 = 2$

$4a = 2$

$a = \frac{1}{2}$

même valeur

$(-2; \frac{25}{2}) \in h \Rightarrow h(-2) = \frac{25}{2}$ $\quad a(-2-3)^2 = \frac{25}{2}$

$a \cdot (-5)^2 = \frac{25}{2}$

$25a = \frac{25}{2} \Rightarrow a = \frac{1}{2}$

$h(x) = \frac{1}{2}(x-3)^2$

d) j est une fonction impaire de degré 3, elle est de la forme :

$j(x) = a_3x^3 + a_1x$

$(1; 2) \in j \Rightarrow a_3 \cdot 1^3 + a_1 \cdot 1 = 2$

$(-\frac{1}{2}; \frac{1}{2}) \in j \Rightarrow a_3\left(-\frac{1}{2}\right)^3 + a_1\left(-\frac{1}{2}\right) = \frac{1}{2}$

$$\Rightarrow \begin{cases} a_3 + a_1 = 2 \\ -\frac{a_3}{8} - \frac{a_1}{2} = \frac{1}{2} \end{cases} \Bigg| \cdot 8 \qquad \begin{aligned} & a_3 + a_1 = 2 \\ + \; & -a_3 - 4a_1 = 4 \\ \hline & -3a_1 = 6 \end{aligned}$$

$\Rightarrow a_1 = -2$

$\Rightarrow a_3 - 2 = 2$

$\Rightarrow a_3 = 4$

$j(x) = 4x^3 - 2x$