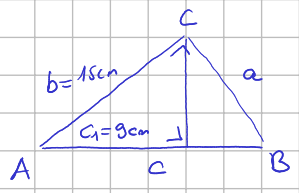
Géométrie: Théorèmes métriques

Exercice 2



Par Pythagore: $h_c^2 = 15^2 - 9^2$

$h_c^2 = 225 - 81$

$h_c^2 = 144 \Rightarrow \boxed{h_c = \sqrt{144} = 12\,cm}$

Par le thm de la hauteur:

$h_c^2 = c_1 \cdot c_2$

$12^2 = 9 \cdot c_2 \Rightarrow \boxed{c_2 = \frac{144}{9} = 16\,cm}$

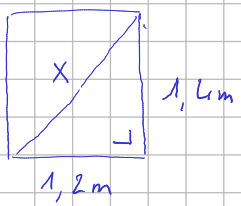
$\boxed{c = c_1 + c_2 = 9 + 16 = 25\,cm}$

Par Pythagore: $a^2 = 25^2 - 15^2$

$a^2 = 625 - 225$

$a^2 = 400 \Rightarrow \boxed{a = \sqrt{400} = 20\,cm}$

Exercice 3



Par Pythagore: $x = \sqrt{1,2^2 + 1,4^2} = \sqrt{3,4} \approx 1,84\,m$

Pour passer la planche, il faut au moins 1,9m.

Non, la planche ne passe pas.

Exercice 4

a) Théorème de la hauteur: $h_c^2 = c_1 \cdot c_2$

$x^2 = 5 \cdot (20-5)$

$x^2 = 5 \cdot 15$

$x^2 = 75$

$x = \sqrt{75} = 5\sqrt{3} = 8,66\,dm$

b) thm de la hauteur: $x^2 = 7 \cdot 15$

$x = \sqrt{105} = 10,25\,cm$

Exercice 4 (suite)

c) thm de la hauteur : $16^2 = 10 \cdot (x-10)$ | CL

$256 = 10x - 100$ | $+100$

$356 = 10x$ | $:10$

$35,6 = x$

$x = 35,6$ mm

d) $x = 20 - 14 = 6$ m

thm de la hauteur : $y^2 = 14 \cdot 6$

$y^2 = 84$

$y = \sqrt{84} = 2\sqrt{21} = 9,17$ m

Pythagore : $z^2 = x^2 + y^2$

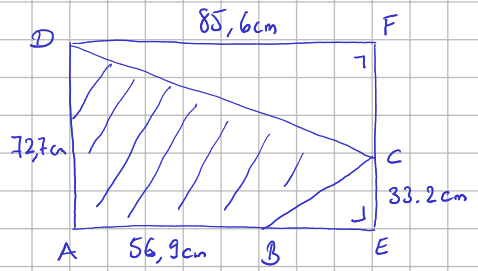
$z^2 = 6^2 + (\sqrt{84})^2$

$z^2 = 36 + 84$

$z^2 = 120$

$z = \sqrt{120} = 2\sqrt{30} = 10,95$ m

Exercice 5



$\overline{BE} = 85,6 - 56,9 = 28,7$ cm

$\overline{FC} = 72,7 - 33,2 = 39,5$ cm

Par Pythagore : $\overline{BC} = \sqrt{\overline{BE}^2 + \overline{CE}^2}$

$= \sqrt{28,7^2 + 33,2^2} = \sqrt{1925,93}$

$\simeq 43,9$ cm

Par Pythagore : $\overline{DC} = \sqrt{\overline{DF}^2 + FC^2} = \sqrt{85,6^2 + 39,5^2} = \sqrt{8887,61} \simeq 94,3$ cm

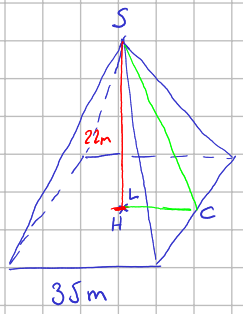
Périmètre : $P = 72,7 + 56,9 + 43,9 + 94,3 = 267,8$ cm

Aire $= A_{\square} - A_{\triangle} - A_{\triangle}$

Aire : $A = 85,6 \cdot 72,7 - \frac{28,7 \cdot 33,2}{2} - \frac{85,6 \cdot 39,5}{2} = 6223,12 - 476,42 - 1690,6$

$= 4056,1$ cm²

Exercice 6



$HC = \frac{35}{2} = 17{,}5\,cm$

Par Pythagore : $\overline{SC}^2 = 22^2 + 17{,}5^2$

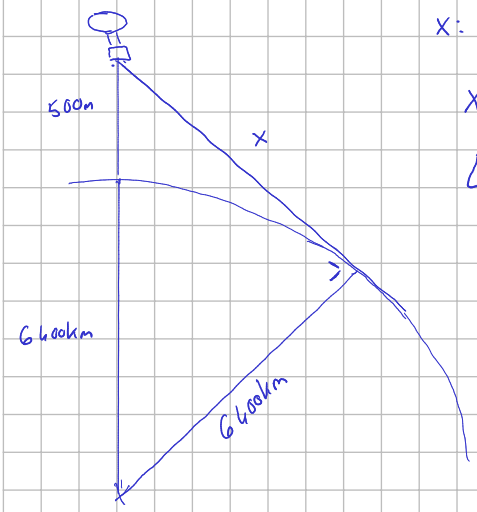
$\overline{SC}^2 = 790{,}25 \Rightarrow \overline{SC} \simeq 28{,}11\,m$

Surface en verre :

$$A = \frac{2 \cancel{4} \cdot 35 \cdot \overline{SC}}{\cancel{2}} = 70 \cdot \overline{SC} \simeq 1967{,}8\,m^2$$

Prix : $280 \cdot A \simeq 550'983{,}16$ CHF

Exercice 7

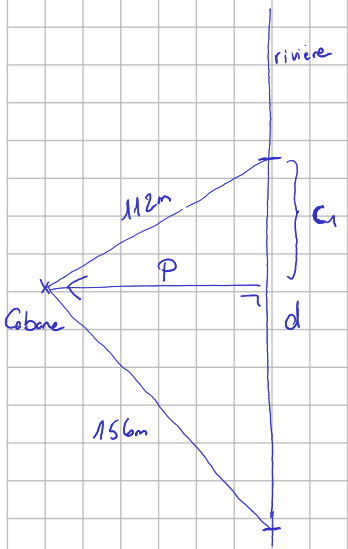


x : distance rongeoffice horizon [km] 500m = 0,5km

$x = \sqrt{6400{,}5^2 - 6400^2} = \sqrt{6400{,}25} \simeq 80\,km$

L'horizon se trouve à ~ 80km.

Exercice 8



a) $d = \sqrt{112^2 + 156^2} = \sqrt{36880} \simeq 192\,m$

Les deux randonneurs se trouvent à 192 m l'un de l'autre.

b) Théorème d'Euclide :

$112^2 = c_1 \cdot d \Rightarrow c_1 = \frac{112^2}{d} = \frac{112^2}{192} = 65{,}3\,m$

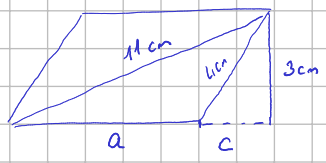
Théorème de Pythagore :

$p = \sqrt{112^2 - c_1^2} = \sqrt{112^2 - 65{,}3^2} \simeq 90{,}98\,m$

La distance entre la cabane et la rivière est de 91m.

Exercice 9

a)



Par Pythagore : $c = \sqrt{4^2 - 3^2} = \sqrt{7} \simeq 2,64$ cm

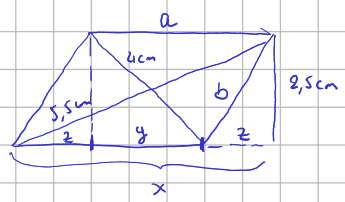
$a+c = \sqrt{11^2 - 3^2} = \sqrt{112} = \sqrt{16 \cdot 7} = 4\sqrt{7} \simeq 10,58$ cm

$a = 4\sqrt{7} - \sqrt{7} = 3\sqrt{7} \simeq 7,94$ cm

$P = 2 \cdot 3\sqrt{7} + 2 \cdot 4 = 6\sqrt{7} + 8 \simeq 23,87$ cm

$A = 3\sqrt{7} \cdot 3 = 9\sqrt{7} = 23,81$ cm²

b)



$x = \sqrt{5,5^2 - 2,5^2} = \sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6} \simeq 4,9$ cm

$y = \sqrt{4^2 - 2,5^2} = \sqrt{9,75} \simeq 3,12$ cm

$z = \dfrac{2\sqrt{6} - \sqrt{9,75}}{2} = 0,89$ cm

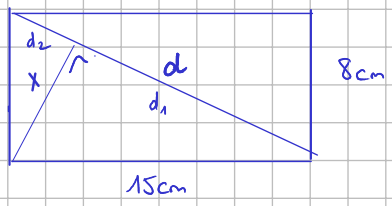
$a = 3,12 + 0,89 = 4,01$ cm

$b = \sqrt{2,5^2 + 0,89^2} = 2,65$ cm

$P = 2(a+b) = 2 \cdot (4,01 + 2,65) = 13,33$ cm

$A = a \cdot h = 4,01 \cdot 2,5 = 10,03$ cm²

Exercice 10



Par Pythagore : $d = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$ cm

thm d'Euclide : $15^2 = d_1 \cdot 17$

$d_1 = \dfrac{15^2}{17} = \dfrac{225}{17} \simeq 13,24$ cm

thm de la hauteur : $x^2 = d_1 \cdot d_2$

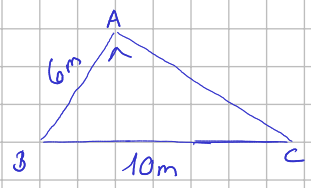
$x^2 = \dfrac{225}{17} \cdot \left(17 - \dfrac{225}{17}\right)$

$x^2 = \dfrac{225}{17} \cdot \dfrac{64}{17}$

$x^2 = \dfrac{14400}{289}$

$x = \sqrt{\dfrac{14400}{289}} = \dfrac{120}{17} = 7,1$ cm

Exercice 11



$\overline{AC} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8\,m$

$$A_{//} = A_{\cap_{BC}} + A_{\cap_{AC}} + A_{\triangle} - A_{\cap_{BC}}$$

$$= \frac{6^2\pi}{2} + \frac{8^2\cdot\pi}{2} + \frac{6\cdot 8}{2} - \frac{10^2\cdot\pi}{2}$$

$$= \frac{36\pi + 64\pi - 100\pi + 48}{2} = \frac{48}{2} = 24\,m^2$$

Périmètre : $P = P_{\cap_{BC}} + P_{\cap_{AC}} + P_{\cap_{BC}}$

$$= \frac{6\cdot\pi}{2} + \frac{8\cdot\pi}{2} + \frac{10\cdot\pi}{2} = 12\pi \simeq 37{,}7\,m$$