

Corrigés: Racines

Exercice 37

$$a) \sqrt{12 \cdot 3} = \sqrt{12 \cdot 3} = \sqrt{36} = 6$$

$$b) \sqrt[3]{2^3 \cdot 4} = \sqrt[3]{2 \cdot 4} = \sqrt[3]{8} = 2$$

$$c) \sqrt{5} \sqrt{\frac{1}{35}} = \sqrt{\frac{5}{35}} = \sqrt{\frac{1}{7}} = \frac{\sqrt{1}}{\sqrt{7}} = \frac{1}{\sqrt{7}}$$

$$d) \sqrt{365^2} = 365$$

$$e) \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$f) \frac{3}{\sqrt{15}} \sqrt{\frac{5}{6}} = 3 \cdot \sqrt{\frac{5}{15 \cdot 6}} = 3 \sqrt{\frac{1}{18}} = 3 \frac{\sqrt{1}}{\sqrt{18}} = \frac{3}{\sqrt{2 \cdot 9}} = \frac{3}{\sqrt{2} \cdot 3} = \frac{\cancel{3}^1}{\sqrt{2} \cdot \cancel{3}_1} = \frac{1}{\sqrt{2}}$$

$$g) \sqrt{\sqrt{16}} = \sqrt{4} = 2$$

$$h) \sqrt[3]{\sqrt{8}} = \sqrt[3]{2^2} = \sqrt[3]{8} = \sqrt{2}$$

$$i) -\sqrt{\frac{28}{5}} \sqrt{\frac{35}{4}} = -\sqrt{\frac{\cancel{28}^7 \cdot \cancel{35}^7}{\cancel{5}_1 \cdot \cancel{4}_1}} = -\sqrt{\frac{49}{1}} = -\sqrt{49} = -7$$

Exercice 38

$$a) \sqrt{12} = \sqrt{3 \cdot 4} = \sqrt{3} \cdot \sqrt{4} = 2\sqrt{3}$$

$$g) \sqrt{500} = \sqrt{100 \cdot 5} = \sqrt{100} \cdot \sqrt{5} = 10\sqrt{5}$$

$$b) \sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$$

$$h) \sqrt{108} = \sqrt{9 \cdot 12} = \sqrt{9 \cdot 4 \cdot 3} = \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{3} = 3 \cdot 2\sqrt{3} = 6\sqrt{3}$$

$$c) \sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$$

$$i) \sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$$

$$d) \sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

$$j) \sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

$$e) \sqrt{147} = \sqrt{49 \cdot 3} = \sqrt{49} \cdot \sqrt{3} = 7\sqrt{3}$$

$$k) \sqrt[3]{511} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$$

$$f) \sqrt{216} = \sqrt{36 \cdot 6} = \sqrt{36} \cdot \sqrt{6} = 6\sqrt{6}$$

$$l) \sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8} \cdot \sqrt[3]{4} = 2\sqrt[3]{4}$$

Exercice 39

$$a) \sqrt{3} \cdot \sqrt{7} \cdot \sqrt{21} = \sqrt{3 \cdot 7 \cdot 21} = \sqrt{3 \cdot 7 \cdot 3 \cdot 7} = \sqrt{3 \cdot 3} \cdot \sqrt{7 \cdot 7} = 3 \cdot 7 = 21$$

$$b) (\sqrt{3})^6 = (3^{\frac{1}{2}})^6 = 3^{\frac{6}{2}} = 3^3 = 27$$

$$c) \sqrt{5} \cdot (\sqrt{20} + \sqrt{5}) = \sqrt{5} \cdot \sqrt{20} + \sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 20} + \sqrt{5 \cdot 5} = \sqrt{100} + 5 = 10 + 5 = 15$$

$$d) \sqrt{45} : \sqrt{5} = \frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{\frac{9 \cdot 5}{5}} = \sqrt{9} = 3$$

$$e) \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} = 3 \cdot 3 = 9$$

Exercice 39 (suite)

$$f) (\sqrt{2})^8 = (2^{\frac{1}{2}})^8 = 2^{\frac{8}{2}} = 2^4 = 16$$

$$g) \sqrt{6 \cdot \left(\sqrt{\frac{1}{3}} + \sqrt{\frac{1}{2}} \right)} = \sqrt{6 \cdot \frac{1}{3}} + \sqrt{6 \cdot \frac{1}{2}} = \sqrt{\frac{6}{3}} + \sqrt{\frac{6}{2}} = \sqrt{2} + \sqrt{3}$$

$$h) \frac{\sqrt{3} \cdot \sqrt{32}}{\sqrt{6}} = \sqrt{\frac{3 \cdot 32}{6}} = \sqrt{16} = 4$$

$$i) \frac{\sqrt{2} \cdot \sqrt{12}}{\sqrt{3} \cdot \sqrt{27}} \cdot \sqrt{2 \cdot \frac{2}{3}} = \frac{\sqrt{2 \cdot 12 \cdot 2 \cdot 2}}{\sqrt{3 \cdot 27 \cdot 3}} = \frac{\sqrt{2 \cdot 4 \cdot 2 \cdot 2}}{\sqrt{27 \cdot 3}} = \frac{\sqrt{2 \cdot 2 \cdot 4 \cdot 2}}{\sqrt{9 \cdot 3 \cdot 3}} = \frac{\sqrt{2 \cdot 2} \sqrt{4 \cdot 2}}{\sqrt{9} \sqrt{3 \cdot 3}} = \frac{2 \cdot 2 \cdot \sqrt{2}}{3 \cdot 3} = \frac{4\sqrt{2}}{9}$$

Exercice 40

$$a) 9^{-2} = \frac{1}{9^2} = \frac{1}{81}$$

$$b) 9^0 = 1$$

$$c) 9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$d) 8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$e) 10^{-5} = \frac{1}{100000}$$

$$f) 16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$$

$$g) 49^{\frac{1}{2}} = \sqrt{49} = 7$$

$$h) 100^0 = 1$$

$$i) 25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

$$j) 4^{\frac{5}{2}} = \sqrt{4^5} = 2^5 = 32$$

$$k) 81^{\frac{1}{4}} = \sqrt[4]{81} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3} = 3$$

$$l) 27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$$

Exercice 41

$$a) a^{\frac{1}{2}} = \sqrt{a}$$

$$b) m^{\frac{1}{3}} = \sqrt[3]{m}$$

$$c) p^{\frac{3}{4}} = \sqrt[4]{p^3} = (\sqrt[4]{p})^3$$

$$d) x^{-\frac{1}{3}} = \frac{1}{x^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{x}}$$

$$e) b^{0,5} = b^{\frac{1}{2}} = \sqrt{b}$$

$$f) (a^2 b^3)^{\frac{2}{4}} = \sqrt[4]{(a^2 b^3)^2} = \sqrt[4]{a^4 b^6}$$

Exercise 42

$$a) \sqrt{a} = a^{\frac{1}{2}}$$

$$b) \sqrt[3]{a^3} = a^{\frac{3}{3}}$$

$$c) \sqrt[4]{a^5} = a^{\frac{5}{4}}$$

$$d) \frac{1}{\sqrt[3]{a}} = \frac{1}{a^{\frac{1}{3}}} = a^{-\frac{1}{3}}$$

$$e) \sqrt[5]{a} = a^{\frac{1}{5}}$$

$$f) \frac{1}{\sqrt[3]{a^2}} = \frac{1}{a^{\frac{2}{3}}} = a^{-\frac{2}{3}}$$

$$g) (\sqrt[4]{a})^3 = a^{\frac{3}{4}}$$

$$h) \frac{1}{\sqrt[4]{a^3}} = \frac{1}{a^{\frac{3}{4}}} = a^{-\frac{3}{4}}$$

$$i) \sqrt[3]{a^2} = a^{\frac{2}{3}}$$

Exercise 43

$$a) \sqrt{a} \cdot a^5 \cdot a^{\frac{2}{5}} = a^{\frac{1}{2}} \cdot a^5 \cdot a^{\frac{2}{5}} = a^{\frac{1}{2} + 5 + \frac{2}{5}} = a^{\frac{2}{5} + \frac{20}{5} + \frac{2}{5}} = a^{\frac{24}{5}} = \sqrt[5]{a^{24}}$$

$$b) \sqrt{x^2} \cdot \sqrt{x^5} \cdot x^{\frac{1}{4}} = x^{\frac{2}{2}} \cdot x^{\frac{5}{2}} \cdot x^{\frac{1}{4}} = x^{\frac{2}{2} + \frac{5}{2} + \frac{1}{4}} = x^{\frac{8}{4} + \frac{10}{4} + \frac{1}{4}} = x^{\frac{19}{4}} = \sqrt[4]{x^{19}}$$

$$c) (\sqrt[3]{b^7})^3 \cdot b^{\frac{2}{3}} = (b^{\frac{7}{3}})^3 \cdot b^{\frac{2}{3}} = b^{\frac{21}{3}} \cdot b^{\frac{2}{3}} = b^{\frac{112}{14}} \cdot b^{\frac{2}{14}} = b^{\frac{114}{14}} = \sqrt[14]{b^{114}}$$

$$d) \frac{\sqrt[5]{x}}{\sqrt[3]{x^2}} \cdot x^{\frac{7}{3}} = \frac{x^{\frac{1}{5}}}{x^{\frac{2}{3}}} \cdot x^{\frac{7}{3}} = x^{\frac{1}{5} - \frac{2}{3} + \frac{7}{3}} = x^{\frac{3}{15} - \frac{10}{15} + \frac{35}{15}} = x^{\frac{28}{15}} = \sqrt[15]{x^{28}}$$

$$e) \sqrt[5]{a} \cdot \sqrt[4]{a} \cdot \sqrt[3]{a} \cdot \sqrt{a} = a^{\frac{1}{5}} \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2}} = a^{\frac{12}{60} + \frac{15}{60} + \frac{20}{60} + \frac{30}{60}} = a^{\frac{77}{60}} = \sqrt[60]{a^{77}}$$

$$f) (b^{\frac{2}{3}} \cdot b^{\frac{1}{4}} \cdot \sqrt[3]{b^5})^2 = (b^{\frac{2}{3}} \cdot b^{\frac{1}{4}} \cdot b^{\frac{5}{3}})^2 = (b^{\frac{8}{12} + \frac{3}{12} + \frac{20}{12}})^2 = (b^{\frac{31}{12}})^2 = b^{\frac{62}{12}} = b^{\frac{31}{6}} = \sqrt[6]{b^{31}}$$

Exercise 44

$$a) \sqrt[3]{\sqrt[3]{64}} = \sqrt[3]{4} = 2$$

$$b) \sqrt[7]{\sqrt[5]{a^{14}}} = \sqrt[35]{a^{14}} = a^{\frac{14}{35}} = a^{\frac{2}{5}}$$

$$c) \sqrt[10]{\sqrt[5]{a^{50}}} = \sqrt[50]{a^{50}} = a$$

$$d) \sqrt[8]{a^{-4}} = a^{-\frac{4}{8}} = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}}$$

$$e) \sqrt[3]{a^2 \cdot \sqrt[3]{a^2}} = (a^2 \cdot a^{\frac{2}{3}})^{\frac{1}{3}} = (a^{2 + \frac{2}{3}})^{\frac{1}{3}} = (a^{\frac{8}{3}})^{\frac{1}{3}} = a^{\frac{8}{3} \cdot \frac{1}{3}} = a^{\frac{8}{9}}$$

$$f) \sqrt{x^4 \cdot \sqrt[3]{x^{\frac{1}{2}}}} = (x^4 \cdot x^{\frac{1}{6}})^{\frac{1}{2}} = (x^{4 + \frac{1}{6}})^{\frac{1}{2}} = (x^{\frac{25}{6}})^{\frac{1}{2}} = x^{\frac{25}{6} \cdot \frac{1}{2}} = x^{\frac{25}{12}}$$

Exercise 45

$$a) \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$b) \frac{25}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{25\sqrt{5}}{5} = 5\sqrt{5}$$

$$c) \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{5} = \frac{\sqrt{15}}{5}$$

$$d) \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

$$e) \frac{1}{3\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{11}}{3 \cdot 11} = \frac{\sqrt{11}}{33}$$

$$f) \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$g) \frac{\sqrt{7}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{7} \cdot \sqrt{6}}{6} = \frac{\sqrt{42}}{6}$$

$$h) \frac{2\sqrt{2}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{2} \cdot \sqrt{5}}{3 \cdot 5} = \frac{2\sqrt{10}}{15}$$

$$i) \frac{2}{3\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{6}}{3 \cdot 6} = \frac{\sqrt{6}}{9}$$

$$j) \frac{4}{3\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{3 \cdot 7} = \frac{4\sqrt{7}}{21}$$

$$k) \frac{15}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{15\sqrt{10}}{2 \cdot 10} = \frac{3\sqrt{10}}{4}$$

$$l) \frac{2\sqrt{5}}{3\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{5} \cdot \sqrt{6}}{3 \cdot 6} = \frac{\sqrt{30}}{9}$$

Exercise 18 - Challenge

$$a) \sqrt[4]{\frac{a^{\frac{1}{3}}}{a^{-5}}} \cdot \sqrt[6]{\frac{a^{\frac{3}{4}}}{a^2}} = \sqrt[12]{\frac{a^{\frac{1}{12}} \cdot a^{-\frac{8}{12}}}{a^{-\frac{5}{3}}}} : \frac{a^{\frac{3}{24}}}{a^{\frac{2}{3}}} = \sqrt[12]{a^{\frac{1}{12} - \frac{48}{12} - (-\frac{10}{12})}} : a^{\frac{3}{24} - \frac{16}{24}}$$

$$= \sqrt[12]{a^{-\frac{27}{12}}} : a^{-\frac{13}{24}} = a^{-\frac{27}{24}} : a^{-\frac{13}{24}} = a^{-\frac{27}{24} - (-\frac{13}{24})} = a^{-\frac{27}{24} + \frac{13}{24}} = a^{-\frac{14}{24}} = a^{-\frac{7}{12}}$$

$$b) \frac{\sqrt[3]{\left(\frac{1}{b^2}\right)^{-4}} \cdot \sqrt{b^3 \sqrt{b^2}}}{b \cdot \sqrt[6]{b \sqrt{b}} \cdot \sqrt[4]{b^{-1} \sqrt[3]{b^2}}} = \frac{\sqrt[3]{(b^{-2})^4} \cdot \sqrt{b \cdot b^{\frac{2}{3}}}}{b \cdot \sqrt[6]{b \cdot b^{\frac{1}{2}}} \cdot \sqrt[4]{b^{-1} \cdot b^{\frac{2}{3}}}} = \frac{\sqrt[3]{b^{\frac{8}{3}}} \cdot \sqrt{b^{\frac{5}{3}}}}{b \cdot \sqrt[6]{b^{\frac{3}{2}}} \cdot \sqrt[4]{b^{-\frac{1}{3}}}}$$

$$= \frac{\sqrt[6]{b^{\frac{8}{3}} \cdot b^{\frac{5}{2}}}}{b \cdot b^{\frac{3}{12}} \cdot b^{-\frac{1}{6}}} = \frac{\sqrt[6]{b^{\frac{21}{6}}}}{b^{\frac{13}{12} + \frac{3}{12} - \frac{2}{12}}} = \frac{b^{\frac{21}{12}}}{b^{\frac{13}{12}}} = b^{\frac{21}{12} - \frac{13}{12}} = b^{\frac{8}{12}} = b^{\frac{2}{3}}$$