

Corrigés: Puissances

Exercice 29

$$\begin{array}{lll}
 \text{a)} \quad x^2 \cdot x^6 = x^{2+6} = x^8 & \text{h)} \quad \left(\frac{6}{6^4}\right) \cdot 6^2 = 6^{-1-4} = 6^{-5} & \text{g)} \quad 4^{-2} \cdot a^{-2} = (4a)^{-2} = (4a)^{-2} \\
 \text{b)} \quad b^7 \cdot b^5 = b^{7+5} = b^{12} & \text{i)} \quad a^{n+1} \cdot a^{n-1} = a^{n+1+n-1} = a^{2n} & \text{p)} \quad (-4)^n \cdot (-3)^n = (-4 \cdot (-3))^n = 12^n \\
 \text{c)} \quad (a^{3n})^{-2} = a^{3n \cdot (-2)} = a^{-6n} & \text{j)} \quad x^{3m+3} \cdot x^{-m+2} = x^{3m+3-m+2} = x^{2m+5} & \text{q)} \quad 10^x \cdot 0,5^x = (10 \cdot 0,5)^x = 5^x \\
 \text{d)} \quad a^2 \cdot a^5 \cdot a^3 = a^{2+5+3} = a^{10} & \text{k)} \quad \frac{x^4}{x^4} = x^{4-4} = x^0 = 1 & \text{r)} \quad 9^{-n} \cdot 3^{-n} = (9 \cdot 3)^{-n} = 3^{-n} \\
 \text{e)} \quad (x^{4n})^{-5} = x^{4n \cdot (-5)} = x^{-20n} & \text{l)} \quad c^4 \cdot c^3 \cdot c^{-5} = c^{4+3-5} = c^2 & \text{s)} \quad 10^m \cdot 5^m = (10 \cdot 5)^m = 2^m \\
 \text{f)} \quad \frac{a^2}{a^5} = a^{2-5} = a^{-3} & \text{m)} \quad (-5)^4)^5 = (-5)^{4 \cdot 5} = (-5)^{20} = 5^{20} & \text{t)} \quad (4c)^5 \cdot (2c)^5 = (4c \cdot 2c)^5 = 2^5 \\
 \text{g)} \quad (b^6)^5 = b^{6 \cdot 5} = b^{30} & \text{n)} \quad 2^n \cdot 5^n = (2 \cdot 5)^n = 10^n & \text{u)} \quad \left(\frac{x}{2}\right)^4 \cdot \left(\frac{x}{6}\right)^4 = \left(\frac{x}{2} \cdot \frac{x}{6}\right)^4 = \left(\frac{x^2}{12}\right)^4 = \frac{x^8}{12^4}
 \end{array}$$

Exercice 30

$$\begin{array}{lll}
 \text{a)} \quad a^{-3} = \frac{1}{a^3} & \text{d)} \quad \frac{2}{x^{-3}} = 2 \cdot x^3 = 2x^3 & \text{g)} \quad \left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 5^2 = 25 \\
 \text{b)} \quad \frac{1}{b^{-1}} = b^1 = b & \text{e)} \quad a^{-2} \cdot c^3 = \frac{1}{a^2} \cdot c^3 = \frac{c^3}{a^2} & \text{h)} \quad \left(\frac{x}{y}\right)^{-4} = \left(\frac{y}{x}\right)^4 = \frac{y^4}{x^4} \\
 \text{c)} \quad 4 \cdot c^{-5} = 4 \cdot \frac{1}{c^5} = \frac{4}{c^5} & \text{f)} \quad \left(\frac{2}{3}\right)^{-1} = \frac{3}{2} & \text{i)} \quad \left(\frac{a^2 b^3}{h^4 m}\right)^{-3} = \frac{a^{2 \cdot (-3)} \cdot b^{3 \cdot (-3)}}{h^{4 \cdot (-3)} \cdot m^{-3}} = \frac{a^{-6} b^{-9}}{h^{-12} m^{-3}} \\
 & & = \frac{h^{12} \cdot m^3}{a^6 \cdot b^9}
 \end{array}$$

Exercice 31

$$\begin{array}{ll}
 \text{a)} \quad (-2)^3 - 5 \cdot 6 + (-3)^2 = -8 - 30 + 9 = -29 \\
 \text{b)} \quad (3+6) - 5 - 2(2+1)^2 = 9 - 5 - 2 \cdot 3^2 = 9 - 5 - 2 \cdot 9 = 9 - 5 - 18 = -14 \\
 \text{c)} \quad \frac{1}{5 \cdot 4^0} + \left(\frac{15}{14}\right)^{-1} \cdot \frac{1}{7} = \frac{1}{5} + \frac{1}{15} \cdot \frac{1}{7} = \frac{1}{5} + \frac{1}{105} = \frac{21}{105} + \frac{1}{105} = \frac{22}{105} \\
 \text{d)} \quad 3 + 2 \cdot 2^2 + 4 \cdot 3 - (6-1)^2 = 3 + 2 \cdot 2^2 + 4 \cdot 3 - 5^2 = 3 + 2 \cdot 4 + 12 - 25 = 3 + 8 + 12 - 25 = -2 \\
 \text{e)} \quad \frac{1}{12} + \left(\frac{2}{3}\right)^{-1} \cdot (-6)^2 = \frac{1}{12} + \frac{3}{2} \cdot 36 = \frac{1}{12} + \frac{108}{2} = \frac{1}{12} + 54 = \frac{1}{12} + \frac{648}{12} = \frac{649}{12} \\
 \text{f)} \quad 2^2 \cdot (2-4)^5 - 2^2 \cdot 4 = 2^2 \cdot (-2)^5 - 2^2 \cdot 4 = 4 \cdot (-32) - 4 \cdot 4 = -128 - 16 = -144 \\
 \text{g)} \quad 1 - 2^2 : \frac{3^{-1} + 6^{-1}}{2} = 1 - 4 : \frac{\frac{1}{3} + \frac{1}{6}}{2} = 1 - 4 : \frac{\frac{2}{6} + \frac{1}{6}}{2} = 1 - 4 : \frac{\frac{3}{6}}{2} = 1 - 4 : \frac{1}{4} \\
 = 1 - 4 : \left(\frac{1}{4}\right) = 1 - 4 \cdot 4 = 1 - 16 = -15
 \end{array}$$

Exercise 32

$$a) a^4 \cdot a^{28} = a^{4+28} = a^{32}$$

$$b) x^{-25} x^{14} = x^{-25+14} = x^{-11}$$

$$c) u^{-4} u^{-7} u = u^{-4-7+1} = u^{-10}$$

$$d) t^{x-4} : t^{x-6} = t^{x-4-(x-6)} = t^{x-4-x+6} = t^2$$

$$e) (-x^6)^{-} = -x^{6 \cdot 2} = -x^{12}$$

$$f) (b^{4n-1})^2 \cdot b^{-n-1} = b^{(4n-1) \cdot 2} \cdot b^{-n-1} = b^{8n-2} \cdot b^{-n-1} = b^{8n-2-n-1} = b^{7n-3}$$

$$g) (m^x n^y)^3 = m^{3x} n^{3y}$$

$$h) (k^3)^5 \cdot k^5 = k^{3 \cdot 5} \cdot k^5 = k^{15} \cdot k^5 = k^{15+5} = k^{20}$$

$$i) (a^x b)^3 \cdot a^x = a^{3x} b^3 \cdot a^x = a^{3x+x} \cdot b^3 = a^{4x} b^3$$

$$j) \frac{h^{-4}}{h^{-7}} = h^{-4-(-7)} = h^{-4+7} = h^3$$

$$k) \frac{n^0}{n^{-4}} = n^{0-(-4)} = n^4$$

$$l) \frac{2^{-4}}{2^{3-6n}} = 2^{5-(3-6n)} = 2^{5-3+6n} = 2^{2+6n} = 2^{6n+2}$$

$$m) \left(\frac{\pi}{2}\right)^{-1} = \frac{2}{\pi}$$

$$n) \frac{54 a^3 y^2}{18 a^4 y^8} = \frac{54}{18} \cdot \frac{a^3}{a^4} \cdot \frac{y^2}{y^8} = 3 \cdot a^{3-4} \cdot y^{2-8} = 3 a^{-1} y^{-6} = 3 a^{-1} y^{-6}$$

$$o) \frac{16 u^{-3} w^{-2}}{128 u^{-5} w^0} = \frac{16}{128} \cdot \frac{u^{-3}}{u^{-5}} \cdot \frac{w^{-2}}{w^0} = \frac{1}{8} \cdot u^{-3-(-5)} \cdot w^{-2-0} = \frac{1}{8} u^2 w^{-2}$$

$$p) \frac{(-a)^6}{(-a)^3} = (-a)^{6-3} = (-a)^3 = -a^3$$

$$q) (a^{-3} b^5)^3 = a^{-3 \cdot 3} b^{5 \cdot 3} = a^{-9} b^{15}$$

$$r) (u^5 v^{-8} x)^{-4} = u^{5 \cdot (-4)} v^{-8 \cdot (-4)} x^{-4} = u^{-20} v^{32} x^{-4}$$

$$s) \left(\frac{3}{4}\right)^{-5} : \left(\frac{3}{4}\right)^{-3} = \left(\frac{3}{4}\right)^{-5-(-3)} = \left(\frac{3}{4}\right)^{-5+3} = \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = \frac{16}{9}$$

$$t) \left(\frac{-2}{5}\right)^3 \cdot \left(\frac{-2}{5}\right)^{-1} = \left(\frac{-2}{5}\right)^{3-(-1)} = \left(\frac{-2}{5}\right)^{3+1} = \left(\frac{-2}{5}\right)^4 = \frac{(-2)^4}{5^4} = \frac{16}{625}$$

Exercise 33

- a) $0,004 \cdot 500 = 4 \cdot 10^{-3} \cdot 5 \cdot 10^2 = 4 \cdot 5 \cdot 10^{-3+2} = 20 \cdot 10^{-1} = 2$
- b) $60'000'000'000 : 2000 = 6 \cdot 10^{10} : (2 \cdot 10^3) = \frac{6}{2} \cdot 10^{10-3} = 3 \cdot 10^7$
- c) $4 \cdot 10^{10} \cdot 4 \cdot 10^{-19} = 4 \cdot 4 \cdot 10^{10-19} = 16 \cdot 10^{-9} = 1,6 \cdot 10 \cdot 10^{-9} = 1,6 \cdot 10^{1-9} = 1,6 \cdot 10^{-8}$
- d) $6 \cdot 10^1 \cdot 7 \cdot 10^{12} \cdot 4 \cdot 10^{-5} = 6 \cdot 7 \cdot 4 \cdot 10^{1+12-5} = 168 \cdot 10^8 = 1,68 \cdot 10^2 \cdot 10^8 = 1,68 \cdot 10^{2+8} = 1,68 \cdot 10^{10}$
- e) $3000 \cdot 0,00000005 \cdot 20000 = 3 \cdot 10^3 \cdot 5 \cdot 10^{-8} \cdot 2 \cdot 10^4 = 3 \cdot 5 \cdot 2 \cdot 10^{3-8+4} = 30 \cdot 10^{-1} = 3 \cdot 10 \cdot 10^{-1} = 3$
- f) $1 \cdot 10^{-5} \cdot 9 \cdot 10^5 \cdot 2 = 9 \cdot 2 \cdot 10^{-5+5} = 18 \cdot 10^0 = 1,8 \cdot 10$
- g) $10 \cdot 10^{-5} \cdot 10^3 \cdot 10^5 \cdot 19 = 19 \cdot 10^{1-5+3+5} = 19 \cdot 10^4 = 1,9 \cdot 10 \cdot 10^4 = 1,9 \cdot 10^{1+4} = 1,9 \cdot 10^5$
- h) $(2 \cdot 10^3)^4 \cdot (9 \cdot 10^5)^2 = 2^4 \cdot 10^{3 \cdot 4} \cdot 9^2 \cdot 10^{5 \cdot 2} = 16 \cdot 10^{12} \cdot 81 \cdot 10^{10} = 1296 \cdot 10^{12+10} = 1296 \cdot 10^{22}$
 $= 1,296 \cdot 10^3 \cdot 10^{22} = 1,296 \cdot 10^{3+22} = 1,296 \cdot 10^{25}$
- i) $200^4 \cdot 1000^2 = (2 \cdot 10^2)^4 \cdot (10^3)^2 = 2^4 \cdot 10^{2 \cdot 4} \cdot 10^{3 \cdot 2} = 2^4 \cdot 10^8 \cdot 10^6 = 16 \cdot 10^{8+6} = 16 \cdot 10^{14}$
 $= 1,6 \cdot 10 \cdot 10^{14} = 1,6 \cdot 10^{1+14} = 1,6 \cdot 10^{15}$
- j) $0,0003^4 : 0,0001^5 = (3 \cdot 10^{-4})^4 : (10^{-4})^5 = 3^4 \cdot 10^{-4 \cdot 4} : 10^{-4 \cdot 5} = 3^4 \cdot 10^{-16} : 10^{-20}$
 $= 81 \cdot 10^{-16-(-20)} = 81 \cdot 10^4 = 8,1 \cdot 10 \cdot 10^4 = 8,1 \cdot 10^5$

Exercise 34

km hm dam m dm cm mm

- a) $400 \text{ dm}^2 = 4 \cdot 10^2 \text{ dm}^2 = 4 \cdot 10^2 \cdot (10^2)^2 \text{ mm}^2 = 4 \cdot 10^6 \text{ mm}^2$
- b) $6 \text{ mm}^3 = 6 : (10^3)^3 \text{ km}^3 = 6 \cdot 10^{-18} \text{ km}^3$
- c) $78\,000\,000 \text{ litres} = 7,8 \cdot 10^7 \text{ litres} = 7,8 \cdot 10^7 \text{ dm}^3 = 7,8 \cdot 10^7 : (10^3)^4 \text{ km}^3$
 $= 7,8 \cdot 10^7 : 10^{12} \text{ km}^3 = 7,8 \cdot 10^{7-12} \text{ km}^3 = 7,8 \cdot 10^{-5} \text{ km}^3$
- d) $9\,000 \text{ cm}^2 = 9 \cdot 10^3 \text{ cm}^2 = 9 \cdot 10^3 : (10^2)^2 \text{ m}^2 = 9 \cdot 10^3 : 10^4 \text{ m}^2 = 9 \cdot 10^{-1} \text{ m}^2 = 0,9 \text{ m}^2$

Exercice 35

vitesse de la lumière = $2,998 \cdot 10^8$ m/s

a) distance terre - soleil = $1,495 \cdot 10^8$ km = $1,495 \cdot 10^8 \cdot 10^3$ m = $1,495 \cdot 10^{11}$ m

$$t = \frac{d}{v} = \frac{1,495 \cdot 10^{11}}{2,998 \cdot 10^8} = 0,4987 \cdot 10^3 = \sim 4,9 \cdot 10^2 \text{ s} \quad \cdot 60 = \sim 8 \text{ min } 19 \text{ s}$$

Il faut ~ 8 minutes et 19 secondes aux rayons du soleil pour atteindre la terre.

b) $t = 9 \text{ ans} = 9 \cdot 365 \cdot 24 \cdot 60 \cdot 60 = 2,8 \cdot 10^8$ s

$$\text{distance Sirius - terre} = v \cdot t = 2,998 \cdot 10^8 \cdot 2,8 \cdot 10^8 = 8,5 \cdot 10^{16} \text{ m} = 8,5 \cdot 10^{13} \text{ km} \quad \cdot 10^3$$

La distance entre Sirius et la terre est de $8,5 \cdot 10^{13}$ km.

c) distance centre Voie lactée - terre = $3 \cdot 10^{17}$ km = $3 \cdot 10^{20}$ m $\cdot 10^3$

$$\text{durée voyage} = \frac{d}{v} = \frac{3 \cdot 10^{20}}{3 \cdot 2,998 \cdot 10^8} = 3,3356 \cdot 10^{11} \text{ s} = \sim 10'577 \text{ années.}$$

Il lui faudrait $\sim 10'577$ années

Exercice 36

Quantité sang : 5 litres = $5 \text{ dm}^3 = 5 \cdot (10^3)^2 \text{ mm}^3 = 5 \cdot 10^6 \text{ mm}^3$

Nb de globules rouges par $\text{mm}^3 = 5 \cdot 10^6$

Nb de globules blancs par $\text{mm}^3 = 7 \cdot 10^3$

a) Nb de globules rouges : $5 \cdot 10^6 \cdot 5 \cdot 10^6 = 25 \cdot 10^{12} = 2,5 \cdot 10^{13}$

Nb de globules blancs : $7 \cdot 10^3 \cdot 5 \cdot 10^6 = 35 \cdot 10^9 = 3,5 \cdot 10^{10}$

Notre corps contient $2,5 \cdot 10^{13}$ globules rouges et $3,5 \cdot 10^{10}$ globules blancs.

b) $3 \mu\text{m} = 3 \cdot 10^{-6} \text{ m}$ (micromètre)

$$\text{hauteur : } 3 \cdot 10^{-6} \cdot 2,5 \cdot 10^{13} = 7,5 \cdot 10^7 \text{ m} = 7,5 \cdot 10^4 \text{ km}$$

La hauteur de la colonne obtenue est de $7,5 \cdot 10^4$ km, ce qui correspond à un peu moins de 2 fois le tour de la terre!