

Corrigés: Polynômes

Exercice 1

$$a) a^2 + b^2 - (a^2 - b^2 + 3ab) - (b^2 + a^2b) = \cancel{a^2} + \cancel{b^2} - \cancel{a^2} + b^2 - 3ab - \cancel{b^2} - a^2b \\ = b^2 - 3ab - a^2b$$

$$b) 5xy + y^2 - (3xy - (-y^2 - xy)) = 5xy + y^2 - (3xy + y^2 + xy) \\ = 5xy + \cancel{y^2} - 3xy - \cancel{y^2} - xy = xy$$

$$c) a + b - c - (- (b - 3c)) = a + b - c + b - 3c = a + 2b - 4c$$

$$d) 3a^2 + a^4 - 5a^2 - 3a^4 + a^0 = -2a^2 - 2a^4 + 1 = -2a^4 - 2a^2 + 1$$

Exercice 2

$$a) (a-x)(b-y) = ab - ay - bx + xy$$

$$b) (a-x)b - y = ab - bx - y$$

$$c) (x-y-z)(2x+3y) = 2x^2 + 3xy - 2xy - 3y^2 - 2xz - 3yz = 2x^2 - 3y^2 + xy - 2xz - 3yz$$

$$d) (4a-5x)(5c+6b)4n = (20ac + 16ab - 25cx - 20bx)4n = 80acn + 16abn - 100cnx - 80bnx$$

$$e) \left(\frac{2}{3}(x+1) - \frac{1}{2}(x-1)\right) = \frac{2}{3}x + \frac{2}{3} - \frac{1}{2}x + \frac{1}{2} = \frac{4}{6}x + \frac{4}{6} - \frac{3}{6}x + \frac{3}{6} = \frac{1}{6}x + \frac{7}{6} = \frac{x}{6} + \frac{7}{6}$$

$$f) \frac{1}{3}(x-3) + (x+3) - \frac{1}{4}(x+4) = \frac{1}{3}x - 1 + x + 3 - \frac{1}{4}x - 1 = \frac{1}{3}x + x - \frac{1}{4}x + 1$$

$$= \frac{4}{12}x + \frac{12}{12}x - \frac{3}{12}x + 1 = \frac{13}{12}x + 1$$

$$g) (4y+6x)(3a-5b) - (2x-6y)(2a+3b) = 12ay - 20by + 18ax - 30bx - (4ax + 6bx - 12ay - 18by)$$

$$= 12ay - 20by + 18ax - 30bx - 4ax - 6bx + 12ay + 18by$$

$$= -2by - 36bx + 14ax + 24ay$$

$$h) (x-5)(3x-1) - (x-6)(x+5) = 3x^2 - x - 15x + 5 - (x^2 - x - 30)$$

$$= 3x^2 - 16x + 5 - x^2 + x + 30 = 2x^2 - 15x + 35$$

$$i) (4+9a-3c)d^5 - (d^3-6b)9ad^2 = 4d^5 + 9ad^5 - 3cd^5 - (9ad^5 - 54abd^2)$$

$$= 4d^5 + \cancel{9ad^5} - 3cd^5 - \cancel{9ad^5} + 54abd^2 = 4d^5 - 3cd^5 + 54abd^2$$

$$j) \left(\frac{x}{2} - \frac{1}{3}\right)^2 = \left(\frac{x}{2}\right)^2 - 2 \cdot \frac{x}{2} \cdot \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 = \frac{x^2}{4} - \frac{x}{3} + \frac{1}{9}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$k) \left(\frac{x}{3} + \frac{2}{5}\right) \left(\frac{x}{3} - \frac{2}{5}\right) = \left(\frac{x}{3}\right)^2 - \left(\frac{2}{5}\right)^2 = \frac{x^2}{9} - \frac{4}{25}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$l) \left(x + \frac{1}{4}\right) \left(\frac{x}{2} + 2\right) = \frac{x^2}{2} + 2x + \frac{x}{8} + \frac{2}{4} = \frac{x^2}{2} + \frac{16x}{8} + \frac{x}{8} + \frac{1}{2} = \frac{x^2}{2} + \frac{17x}{8} + \frac{1}{2}$$

Exercise 3

$$a) 3a(a^2-1) - 4a^2(a+2) - 3a + 4(a^2-1) =$$

$$3a^3 - 3a - 4a^3 - 8a^2 - 3a + 4a^2 - 4 = -a^3 - 4a^2 - 6a - 4$$

$$b) -3x^2(x^3-2x^2+4) + 4x^3(1-2x) + x(x-1) + 2x =$$

$$-3x^5 + 6x^4 - 12x^2 + 4x^3 - 8x^4 + x^2 - x + 2x = -3x^5 - 2x^4 + 4x^3 - 11x^2 + x$$

$$c) [x - (x^2+3)]2x^2 + 3(x-2) + 4x^2 = [x-x^2-3]2x^2 + 3(x-2) + 4x^2 =$$

$$2x^3 - 2x^4 - 6x^2 + 3x - 6 + 4x^2 = -2x^4 + 2x^3 - 2x^2 + 3x - 6$$

$$d) 3x\{x - [2x^2 - (-x+4)] + 3\} - 3x^2(x-2) =$$

$$3x\{x - [2x^2 + x - 4] + 3\} - 3x^2(x-2) =$$

$$3x\{x - 2x^2 - x + 4 + 3\} - 3x^2(x-2) = 3x\{-2x^2 + 7\} - 3x^2(x-2) =$$

$$-6x^3 + 21x - 3x^3 + 6x^2 = -9x^3 + 6x^2 + 21x$$

$$e) 3a[ab - (b^2+a^2)] - 2b\{2a^2 - [3ab+b^2] + ab\} =$$

$$3a[ab - b^2 - a^2] - 2b\{2a^2 - 3ab - b^2 + ab\} =$$

$$3a[ab - b^2 - a^2] - 2b\{2a^2 - 2ab - b^2\} =$$

$$3a^2b - 3ab^2 - 3a^3 - 4a^2b + 4ab^2 + 2b^3 = -3a^3 - a^2b + ab^2 + 2b^3$$

Exercise 4

$$A = \frac{1}{2}x^3 - \frac{2}{3}x^2 + 1$$

$$B = x^3 - \frac{4}{5}x$$

$$C = \frac{5}{2}x - \frac{2}{5}$$

$$a) P = AB = \left(\frac{1}{2}x^3 - \frac{2}{3}x^2 + 1\right) \left(x^3 - \frac{4}{5}x\right)$$

$$= \frac{1}{2}x^6 - \frac{4}{10}x^4 - \frac{2}{3}x^5 + \frac{8}{15}x^3 + x^3 - \frac{4}{5}x = \frac{1}{2}x^6 - \frac{2}{5}x^4 - \frac{2}{3}x^5 + \frac{8}{15}x^3 + \frac{15}{15}x^3 - \frac{4}{5}x$$

$$= \frac{1}{2}x^6 - \frac{2}{3}x^5 - \frac{2}{5}x^4 + \frac{23}{15}x^3 - \frac{4}{5}x$$

$$b) Q = BC = \left(x^3 - \frac{4}{5}x\right) \left(\frac{5}{2}x - \frac{2}{5}\right)$$

$$= \frac{5}{2}x^4 - \frac{2}{5}x^3 - \frac{2}{5} \cdot \frac{4}{5}x^2 + \frac{8}{25}x = \frac{5}{2}x^4 - \frac{2}{5}x^3 - \frac{2}{5}x^2 + \frac{8}{25}x$$

$$c) R = C(A+B) = \left(\frac{5}{2}x - \frac{2}{5}\right) \left[\left(\frac{1}{2}x^3 - \frac{2}{3}x^2 + 1\right) + \left(x^3 - \frac{4}{5}x\right)\right]$$

$$= \left(\frac{5}{2}x - \frac{2}{5}\right) \left(\frac{1}{2}x^3 - \frac{2}{3}x^2 + 1 + x^3 - \frac{4}{5}x\right)$$

$$= \left(\frac{5}{2}x - \frac{2}{5}\right) \left(\frac{3}{2}x^3 - \frac{2}{3}x^2 + 1 - \frac{4}{5}x\right)$$

$$= \frac{15}{4}x^4 - \frac{5}{3}x^3 + \frac{5}{2}x - 2x^2 - \frac{3}{5}x^3 + \frac{4}{15}x^2 - \frac{2}{5} + \frac{8}{25}x$$

$$= \frac{15}{4}x^4 - \frac{5}{3}x^3 + \frac{5}{2}x - 2x^2 - \frac{3}{5}x^3 + \frac{4}{15}x^2 - \frac{2}{5} + \frac{8}{25}x$$

$$= \frac{15}{4}x^4 - \frac{25}{15}x^3 - \frac{9}{15}x^3 - \frac{30}{15}x^2 + \frac{4}{15}x^2 + \frac{125}{50}x + \frac{16}{50}x - \frac{2}{5}$$

$$= \frac{15}{4}x^4 - \frac{34}{15}x^3 - \frac{26}{15}x^2 + \frac{141}{50}x - \frac{2}{5}$$

$$d) S = 6A - 5B = 6\left(\frac{1}{2}x^3 - \frac{2}{3}x^2 + 1\right) - 5\left(x^3 - \frac{4}{5}x\right)$$

$$= 3x^3 - 4x^2 + 6 - 5x^3 + 4x = -2x^3 - 4x^2 + 4x + 6$$

Exercice 6

a) Volume :

$$V = (50 - 2x)(25 - x) \cdot x$$

$$= (1250 - 50x - 50x + 2x^2) \cdot x$$

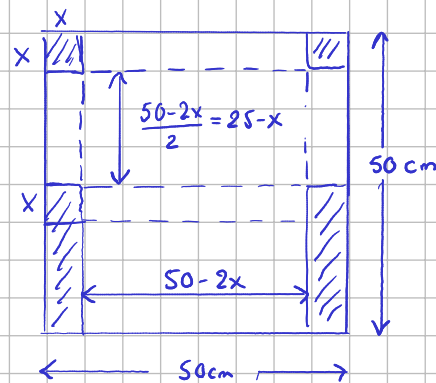
$$= (2x^2 - 100x + 1250) \cdot x$$

$$= 2x^3 - 100x^2 + 1250x$$

b) $x = 2 \text{ cm}$

$$V = 2 \cdot 2^3 - 100 \cdot 2^2 + 1250 \cdot 2 = 2 \cdot 8 - 100 \cdot 4 + 1250 \cdot 2 = 16 - 400 + 2500 = 2116 \text{ cm}^3$$

Le volume vaudra 2116 cm^3 .



Exercise 7

$$a) \quad \begin{array}{r|l} 6x^3 - 17x^2 + 14x - 3 & 2x - 3 \\ - (6x^3 - 9x^2) & 3x^2 - 4x + 1 \end{array} \quad \left| \quad (6x^3 - 17x^2 + 14x - 3) : (2x - 3) \right. = 3x^2 - 4x + 1 \quad \text{reste } 0$$

$$- 8x^2 + 14x$$

$$- (-8x^2 + 12x)$$

$$2x - 3$$

$$- (2x - 3)$$

$$0$$

$$b) \quad (135a^2 + 77 - 30a + 26a^3) : (7 - 4a + 13a^2) = 2a + 11 \quad \text{reste } 0$$

$$\begin{array}{r|l} 26a^3 + 135a^2 - 30a + 77 & 13a^2 - 4a + 7 \\ - (26a^3 - 8a^2 + 14a) & 2a + 11 \end{array}$$

$$143a^2 - 44a + 77$$

$$- (143a^2 - 44a + 77)$$

$$0$$

$$c) \quad (6x^5 + 5x^4 - 25x^3 + 31x^2 - 12x + 5) : (2x^2 - 3x + 2) = 3x^3 + 7x^2 - 5x + 1 \quad \text{reste } x + 3$$

$$\begin{array}{r|l} 6x^5 + 5x^4 - 25x^3 + 31x^2 - 12x + 5 & 2x^2 - 3x + 2 \\ - (6x^5 - 9x^4 + 6x^3) & 3x^3 + 7x^2 - 5x + 1 \end{array}$$

$$14x^4 - 31x^3 + 31x^2$$

$$- (14x^4 - 21x^3 + 14x^2)$$

$$- 10x^3 + 17x^2 - 12x$$

$$- (-10x^3 + 15x^2 - 10x)$$

$$2x^2 - 2x + 5$$

$$- (2x^2 - 3x + 2)$$

$$x + 3$$

$$d) (3m^4 + 6 - 2m^2 + m^3) : (m^2 + 2m) = 3m^2 - 5m + 8 \quad \text{reste } -16m + 6$$

$$\begin{array}{r}
 3m^4 + m^3 - 2m^2 + 6 \quad | \quad m^2 + 2m \\
 \underline{-(3m^4 + 6m^3)} \\
 -5m^3 - 2m^2 + 6 \\
 \underline{-(-5m^3 - 10m^2)} \\
 8m^2 + 6 \\
 \underline{-(8m^2 + 16m)} \\
 -16m + 6
 \end{array}$$

$$e) x^4 : (x+1) = x^3 - x^2 + x - 1 \quad \text{reste } 1$$

$$\begin{array}{r}
 x^4 \quad | \quad x+1 \\
 \underline{-(x^4 + x^3)} \\
 -x^3 \\
 \underline{-(-x^3 - x^2)} \\
 x^2 \\
 \underline{-(x^2 + x)} \\
 -x \\
 \underline{-(-x - 1)} \\
 1
 \end{array}$$

Exercise 8

$$a) (x^3 - 8x^2 + 16x + 4) : (x-5) = x^2 - 3x + 1 \quad \text{reste } 9$$

$$\begin{array}{r}
 x^3 - 8x^2 + 16x + 4 \quad | \quad x-5 \\
 \underline{-(x^3 - 5x^2)} \\
 -3x^2 + 16x + 4 \\
 \underline{-(-3x^2 + 15x)} \\
 x + 4 \\
 \underline{-(x - 5)} \\
 9
 \end{array}$$

$$b) (x^5 - 8x^3 + 8x^2 - 7x + 6) : (x^2 + 3x - 2) = x^3 - 3x^2 + 3x - 7 \quad \text{reste } 20x - 8$$

$$\begin{array}{r} x^5 - 8x^3 + 8x^2 - 7x + 6 \quad | \quad x^2 + 3x - 2 \\ - (x^5 + 3x^4 - 2x^3) \\ \hline -3x^4 - 6x^3 + 8x^2 \\ - (-3x^4 - 9x^3 + 6x^2) \\ \hline 3x^3 + 2x^2 - 7x \\ - (3x^3 + 9x^2 - 6x) \\ \hline -7x^2 - x + 6 \\ - (-7x^2 - 21x + 14) \\ \hline 20x - 8 \end{array}$$

$$c) (5x^3 - 2x^2 + 4x - 4) : (x - 1) = 5x^2 + 3x + 7 \quad \text{reste } 3$$

$$\begin{array}{r} 5x^3 - 2x^2 + 4x - 4 \quad | \quad x - 1 \\ - (5x^3 - 5x^2) \\ \hline 3x^2 + 4x \\ - (3x^2 - 3x) \\ \hline 7x - 4 \\ - (7x - 7) \\ \hline 3 \end{array}$$

$$d) (2x^3 + 7x^2 + 10x + 8) : (2x^3 + 7x) = 1 \quad \text{reste } 7x^2 + 3x + 8$$

$$\begin{array}{r} 2x^3 + 7x^2 + 10x + 8 \quad | \quad 2x^3 + 7x \\ - (2x^3 \quad + 7x) \\ \hline 7x^2 + 3x + 8 \end{array}$$

$$e) (3x^2 - 4x + 12) : (3x^2 - 4x + 1) = 1 \quad \text{reste } 11$$

$$\begin{array}{r} 3x^2 - 4x + 12 \quad | \quad 3x^2 - 4x + 1 \\ - (3x^2 - 4x + 1) \\ \hline 11 \end{array}$$

$$f) (x^{12} - 1) : (x^4 - 1) = x^8 + x^4 + 1$$

$$\begin{array}{r} x^{12} - 1 \quad | \quad x^4 - 1 \\ - (x^{12} - x^8) \\ \hline x^8 - 1 \\ - (x^8 - x^4) \\ \hline x^4 - 1 \\ - (x^4 - 1) \\ \hline 0 \end{array}$$

Challenge

$$a) \left(\frac{5}{8}x^2 - \frac{7}{12}x - \frac{1}{3} \right) : \left(\frac{5}{4}x + \frac{1}{2} \right) = \frac{1}{2}x - \frac{2}{3}$$

$$\begin{array}{r} \frac{5}{8}x^2 - \frac{7}{12}x - \frac{1}{3} \\ - \left(\frac{5}{8}x^2 + \frac{1}{4}x \right) \\ \hline -\frac{10}{12}x - \frac{1}{3} \\ - \left(-\frac{10}{12}x - \frac{1}{3} \right) \\ \hline 0 \end{array} \quad \begin{array}{r} \frac{5}{4}x + \frac{1}{2} \\ \hline \frac{1}{2}x - \frac{2}{3} \end{array}$$

$$-\frac{7}{12}x - \frac{1}{4}x = -\frac{7}{12}x - \frac{3}{12}x = -\frac{10}{12}x$$

$$b) \left(-\frac{1}{3}x^4 + 2x^2 - \frac{1}{2} \right) : \left(\frac{3}{2}x^2 + \frac{1}{3} \right) = -\frac{2}{9}x^2 + \frac{112}{81}$$

$$\text{reste } -\frac{467}{486}$$

$$\begin{array}{r} -\frac{1}{3}x^4 + 2x^2 - \frac{1}{2} \\ - \left(-\frac{1}{3}x^4 - \frac{2}{27}x^2 \right) \\ \hline \frac{56}{27}x^2 - \frac{1}{2} \\ - \left(\frac{56}{27}x^2 + \frac{112}{243} \right) \\ \hline -\frac{467}{486} \end{array} \quad \begin{array}{r} \frac{3}{2}x^2 + \frac{1}{3} \\ \hline -\frac{2}{9}x^2 + \frac{112}{81} \end{array}$$

$$x \cdot \frac{3}{2} = -\frac{1}{3} \Rightarrow x = -\frac{1}{3} : \frac{3}{2} = -\frac{1}{3} \cdot \frac{2}{3} = -\frac{2}{9}$$

$$2x^2 + \frac{2}{27}x^2 = \frac{54}{27}x^2 + \frac{2}{27}x^2 = \frac{56}{27}x^2$$

$$y \cdot \frac{3}{2} = \frac{56}{27} \Rightarrow y = \frac{56}{27} : \frac{3}{2} = \frac{56}{27} \cdot \frac{2}{3} = \frac{112}{81}$$

$$\frac{112}{81} \cdot \frac{1}{3} = \frac{112}{243}$$

$$-\frac{1}{2} - \frac{112}{243} = -\frac{243}{486} - \frac{224}{486} = -\frac{467}{486}$$