

Corrigé: Inéquations

Exercice 31

$$\begin{array}{lcl} \text{a)} & \frac{x}{5} + 8 \leq 10 & | -8 \\ & \frac{x}{5} \leq 2 & | \cdot 5 \\ & x \leq 10 & \\ & S =]-\infty; 10] & \end{array}$$

$$\begin{array}{lcl} \text{b)} & 12x - 36 > 24 & | +36 \\ & 12x > 60 & | :12 \\ & x > 5 & \\ & S =]5; +\infty[& \end{array}$$

$$\begin{array}{lcl} \text{c)} & 3x - 2 + \frac{x}{2} + \frac{13}{2} < \frac{1}{4}x + 12 + 7x & | \cdot 4 \\ & 12x - 8 + 2x + 26 < x + 48 + 28x & | \text{CL} \\ & 14x + 18 < 29x + 48 & | -29x - 18 \\ & -15x < 30 & | : (-15) \\ & x > -2 & \\ & S =]-2; +\infty[& \end{array}$$

$$\begin{array}{lcl} \text{d)} & \frac{1}{2} + \frac{x}{3} - \frac{1}{6} \geq \frac{x}{2} + \frac{1}{3} - \frac{1}{6}x & | \cdot 6 \\ & 3 + 2x - 1 \geq 3x + 2 - x & | \text{CL} \\ & 2x + 2 \geq 2x + 2 & | -2x \\ & 2 \geq 2 \checkmark & \\ & S = \mathbb{R} & \end{array}$$

$$\begin{array}{lcl} \text{e)} & 10\left(\frac{3x-2}{6}\right) - 15\left(\frac{x}{2} + 5\right) \leq 0 & | \text{CL} \\ & \frac{30x}{4} - 20 - \frac{15x}{2} - 75 \leq 0 & | \cdot 4 \\ & 30x - 80 - 30x - 300 \leq 0 & | \text{CL} \\ & -380 \leq 0 \checkmark & \\ & S = \mathbb{R} & \end{array}$$

$$\begin{array}{lcl} \text{f)} & 11(4x+3) + 9 \geq 3(1-6x) + 70 & | \text{CL} \\ & 44x + 33 + 9 \geq 3 - 18x + 70 & | \text{CL} \\ & 44x + 42 \geq -18x + 73 & | +18x - 42 \\ & 62x \geq 31 & | :62 \\ & x \geq \frac{31}{62} & \\ & x \geq \frac{1}{2} & \\ & S = \left[\frac{1}{2}; +\infty\right[& \end{array}$$

Exercice 31 (suite)

$$\begin{array}{lcl}
 g) \quad \frac{2}{9}(8x-3) + \frac{3}{5}(4x + \frac{7}{2}) < 3 & | & CL \\
 \frac{16x}{9} - \frac{6}{9} + \frac{12x}{5} + \frac{21}{10} < 3 & | & \cdot 90 \\
 160x - 60 + 216x + 189 < 270 & | & CL \\
 376x + 129 < 270 & | & -129 \\
 376x < 141
 \end{array}$$

$$x < \frac{141}{376}$$

$$x < \frac{3}{8}$$

$$S =]-\infty; \frac{3}{8}[$$

$$\begin{array}{lcl}
 h) \quad 17x - 8 > 3x + 2(7x - 2) & | & CL \\
 17x - 8 > 3x + 14x - 4 & | & CL \\
 17x - 8 > 17x - 4 & | & -17x \\
 -8 > -4 & \text{faux} & S = \emptyset
 \end{array}$$

Exercice 32

$$\begin{array}{lcl}
 a) \quad 7x + 10 < 2x - 5 & | & -2x - 10 \\
 5x < -15 & | & :5 \\
 x < -3 & S =]-\infty; -3[
 \end{array}$$

$$\begin{array}{lcl}
 b) \quad -3 - \frac{3}{4}x \leq \frac{1}{4}x & | & \cdot 4 \\
 -12 - 3x \leq x & | & +3x \\
 -12 \leq 4x & | & :4 \\
 -3 \leq x & S = [-3; +\infty[
 \end{array}$$

$$\begin{array}{lcl}
 c) \quad -14 - \frac{5}{4}x > -\frac{3}{8}x - 21 & | & \cdot 8 \\
 -112 - 10x > -3x - 168 & | & +3x + 112 \\
 -7x > -56 & | & :(-7) \\
 x < 8 & S =]-\infty; 8[
 \end{array}$$

$$\begin{array}{lcl}
 d) \quad 0,6 + 0,8x > -0,5 + x & | & -0,8x + 0,5 \\
 1,1 > 0,2x & | & :0,2 \\
 5,5 > x & S =]-\infty; 5,5[
 \end{array}$$

$$\begin{array}{lcl}
 e) \quad -\frac{1}{4}(4 + 8x) \leq \frac{1}{2}(x-1) & | & \cdot 4 \\
 -(4 + 8x) \leq 2(x-1) & | & CL \\
 -4 - 8x \leq 2x - 2 & | & -2x + 4 \\
 -10x \leq 2 & | & :(-10) \\
 x \geq -0,2 & S = [-0,2; +\infty[
 \end{array}$$

$$\begin{array}{lcl}
 f) \quad 22x - 21 - 20x < -13 + 20 - 5x & | & CL \\
 2x - 21 < 7 - 5x & | & +5x + 21 \\
 7x < 28 & | & :7 \\
 x < 4 & S =]-\infty; 4[
 \end{array}$$

Exercice 32 (suite)

$$\begin{array}{lcl}
 g) & 2 - 7x + 5(1+x) > 4(1-2x) + 3 & | \text{ CL} \\
 & 2 - 7x + 5 + 5x > 4 - 8x + 3 & | \text{ CL} \\
 & -2x + 7 > 7 - 8x & | +8x - 7 \\
 & 6x > 0 & | : 6 \\
 & x > 0 & S =]0; +\infty[
 \end{array}$$

$$\begin{array}{lcl}
 h) & 23x - 45 + 55 - 25x \geq 0 & | \text{ CL} \\
 & -2x + 10 \geq 0 & | +2x \\
 & 10 \geq 2x & | : 2 \\
 & 5 \geq x & S =]-\infty; 5]
 \end{array}$$

$$\begin{array}{lcl}
 i) & -3(1+3x) \leq 12x - 24x - 18 - 3 & | \text{ CL} \\
 & -3 - 9x \leq -12x - 21 & | +12x + 3 \\
 & 3x \leq -18 & | : 3 \\
 & x \leq -6 & S =]-\infty; -6]
 \end{array}$$

$$\begin{array}{lcl}
 j) & (x-5)^2 \leq (x-7)(x-3) & \\
 & x^2 - 10x + 25 \leq x^2 - 10x + 21 & | -x^2 \\
 & -10x + 25 \leq -10x + 21 & | +10x \\
 & 25 \leq 21 & \text{ faux} \\
 & S = \emptyset
 \end{array}$$

$$\begin{array}{lcl}
 k) & x + \frac{1}{3} \geq 0 & | -\frac{1}{3} \\
 & x \geq -\frac{1}{3} & S = [-\frac{1}{3}; +\infty[
 \end{array}$$

$$\begin{array}{lcl}
 l) & \frac{8+2x}{4} < 2 \cdot \frac{1-x}{-2} & | \text{ CL} \\
 & \frac{8+2x}{4} < -(1-x) & | \cdot 4 \\
 & 8+2x < -4(1-x) & | \text{ CL} \\
 & 8+2x < -4+4x & | -4x - 8 \\
 & -2x < -12 & | : (-2) \\
 & x > 6 & S =]6; +\infty[
 \end{array}$$

Exercice 33

a) Soit x le nombre de boîtes de conserve mises dans le carton.

$$12 \text{ kg} = 12'000 \text{ g}$$

$$\begin{array}{lcl}
 300 \cdot x + 500 & \leq & 12'000 \\
 300x & \leq & 11'500 \\
 x & \leq & \frac{11'500}{300} = \frac{115}{3} \approx 38,3
 \end{array}
 \quad \begin{array}{l} -500 \\ : 300 \end{array}$$

$$S =]-\infty; \frac{115}{3}]$$

On peut mettre au plus 38 boîtes de conserve dans le carton.

Exercice 9 (suite)

b) Soit x le nombre d'heures où l'arroseur est encore allumé.

$$\begin{array}{rcl} 2 \cdot 400 + x \cdot 400 & \geq & 1800 \\ 800 + 400x & \geq & 1800 \\ 400x & \geq & 1000 \end{array} \quad \begin{array}{l} \text{CL} \\ - 800 \\ : 400 \end{array}$$

$$x \geq \frac{10}{4} = \frac{5}{2} = 2,5 \quad S = \left[\frac{5}{2}; +\infty[$$

L'arroseur doit rester encore allumé au moins 2,5 heures.

c) Soit x le nombre de kWh utilisés.

$$\begin{array}{rcl} 0,22 \cdot x + 18 & \geq & 0,18x + 25 \\ 0,04x & \geq & 7 \\ x & \geq & 175 \end{array} \quad \begin{array}{l} - 0,18x - 18 \\ : 0,04 \end{array} \quad S = [175; +\infty[$$

Le tarif B est meilleur marché à partir de 175 kWh

d) Soit x le nombre de courses

$$\begin{array}{rcl} 10,8 \cdot x & \geq & 5,4 \cdot x + 150 \\ 5,4x & \geq & 150 \\ x & \geq & 27,7 \end{array} \quad \begin{array}{l} - 5,4x \\ : 5,4 \end{array} \quad S = [27,7; +\infty[$$

L'abonnement demi-tarif est avantageux à partir de 28 courses.

e) Soit x le montant du dommage [CHF]

$$\begin{array}{rcl} \frac{80}{100} \cdot x - 300 & \geq & \frac{75}{100}x - 200 \\ 80x - 30000 & \geq & 75x - 20000 \\ 5x & \geq & 10000 \\ x & \geq & 2000 \end{array} \quad \begin{array}{l} \cdot 100 \\ - 75x + 30000 \\ : 5 \end{array} \quad S = [2000; +\infty[$$

L'assuré a avantage à choisir l'assurance AP1a, si le montant du dommage est supérieur à 2000 CHF.

Exercice 34

$$a) \frac{x-4}{2} < \frac{7x}{2} - (3x+2) \quad | \cdot 2$$

$$x-4 < 7x - 2(3x+2) \quad | \text{CL}$$

$$x-4 < 7x - 6x - 4 \quad | \text{CL}$$

$$x-4 < x-4 \quad | -4$$

$$x < x \quad \text{faux} \quad S = \emptyset$$

$$b) \frac{x-5}{3} - \frac{x-8}{4} \leq 0 \quad | \cdot 12$$

$$4(x-5) - 3(x-8) \leq 0 \quad | \text{CL}$$

$$4x - 20 - 3x + 24 \leq 0$$

$$x + 4 \leq 0$$

$$x \leq -4 \quad S =]-\infty; -4]$$

$$c) \frac{3t}{2} - \frac{2t}{3} \geq 5\left(\frac{t}{6} + 1\right) - 5 \quad | \text{CL}$$

$$\frac{3t}{2} - \frac{2t}{3} \geq \frac{5t}{6} + 5 - 5 \quad | \text{CL}$$

$$\frac{3t}{2} - \frac{2t}{3} \geq \frac{5t}{6} \quad | \cdot 6$$

$$9t - 4t \geq 5t \quad | \text{CL}$$

$$5t \geq 5t \quad | -5t$$

$$0 \geq 0 \quad \text{Vrai} \quad S = \mathbb{R}$$

$$d) \frac{5y+1}{3} - \frac{8y+1}{4} > \frac{10y+1}{12} \quad | \cdot 12$$

$$4(5y+1) - 3(8y+1) > 10y+1 \quad | \text{CL}$$

$$20y + 4 - 24y - 3 > 10y + 1 \quad | \text{CL}$$

$$-4y + 1 > 10y + 1 \quad | -10y - 1$$

$$-14y > 0 \quad | : (-14)$$

$$y < 0$$

$$S =]-\infty; 0[$$

$$e) -3 < 2z - 5 < 7 \quad | +5$$

$$2 < 2z < 12 \quad | : 2$$

$$1 < z < 6$$

$$S =]1; 6[$$

$$f) 4 \geq 3x + 5 > -1 \quad | -5$$

$$-1 \geq 3x > -6 \quad | : 3$$

$$-\frac{1}{3} \geq x > -2$$

$$S =]-2; -\frac{1}{3}]$$

$$g) 11 > 5(3y-2) > -11 \quad | \text{CL}$$

$$11 > 15y - 10 > -11 \quad | +10$$

$$21 > 15y > -1 \quad | : 15$$

$$\frac{21}{15} > y > -\frac{1}{15}$$

$$S =]-\frac{1}{15}; \frac{7}{5}[$$

$$h) 3 \leq \frac{2x-3}{5} < 7 \quad | \cdot 5$$

$$15 \leq 2x - 3 < 35 \quad | +3$$

$$18 \leq 2x < 38 \quad | : 2$$

$$9 \leq x < 19$$

$$S = [9; 19[$$

Exercice 10 (suite)

$$\begin{array}{l}
 i) \quad -2 < \frac{4u+1}{3} \leq 0 \\
 -6 < 4u+1 \leq 0 \\
 -7 < 4u \leq -1 \\
 -\frac{7}{4} < u \leq -\frac{1}{4} \\
 S =]-\frac{7}{4}; -\frac{1}{4}]
 \end{array}
 \left| \begin{array}{l} \cdot 3 \\ -1 \\ : 4 \end{array} \right.$$

$$\begin{array}{l}
 j) \quad (x+2)(x-3) < (x+5)(x-4) \\
 x^2 - x - 6 < x^2 + x - 20 \\
 -x - 6 < x - 20 \\
 -2x < -14 \\
 x > 7 \\
 S =]7; +\infty[
 \end{array}
 \left| \begin{array}{l} CL \\ -x^2 \\ -x+6 \\ : (-2) \\ \underline{\underline{7}} \end{array} \right.$$

$$\begin{array}{l}
 k) \quad (3y-1)(y+2) \geq 3y^2 + 9y - 2 \\
 3y^2 - y + 6y - 2 \geq 3y^2 + 9y - 2 \\
 5y - 2 \geq 9y - 2 \\
 0 \geq 4y \\
 0 \geq y \\
 S =]-\infty; 0]
 \end{array}
 \left| \begin{array}{l} CL \\ -3y^2 \\ -5y+2 \\ : 4 \end{array} \right.$$

$$\begin{array}{l}
 l) \quad (t-3)^2 \leq (t-4)(t-2) \\
 t^2 - 6t + 9 \leq t^2 - 6t + 8 \\
 -6t + 9 \leq -6t + 8 \\
 9 \leq 8 \quad \text{faux} \\
 S = \emptyset
 \end{array}
 \left| \begin{array}{l} CL \\ -t^2 \\ +6t \end{array} \right.$$

$$\begin{array}{l}
 m) \quad (2x-3)(4x+5) \leq (8x+1)(x-7) \\
 8x^2 - 12x + 10x - 15 \leq 8x^2 + x - 56x - 7 \\
 -2x - 15 \leq -55x - 7 \\
 53x \leq 8 \\
 x \leq \frac{8}{53} \\
 S =]-\infty; \frac{8}{53}]
 \end{array}
 \left| \begin{array}{l} -8x^2 \\ +55x+15 \\ : 53 \end{array} \right.$$