

Corrigés: Inéquations quadratiques

Exercice 32

a) $(x-2)(x+6) < 0$

racines $x_1 = 2$ $x_2 = -6$

		-6		2	
$(x-2)$	-	-	-	0	+
$(x+6)$	-	0	+	+	+
$P(x)$	+	0	-	0	+

$S =]-6; 2[$

b) $(2-5x)(x-3) \geq 0$

racines $x_1 = \frac{2}{5}$ $x_2 = 3$

		$\frac{2}{5}$		3	
$2-5x$	+	0	-	-	-
$x-3$	-	-	-	0	+
$P(x)$	-	0	+	0	-

$S = \left[\frac{2}{5}; 3\right]$

c) $(x-2)(x+4)(1-x) \geq 0$

racines $x_1 = 2$ $x_2 = -4$ $x_3 = 1$

		-4		1		2	
$x-2$	-	-	-	-	0	+	
$x+4$	-	0	+		+		+
$1-x$	+		+	0	-		-
$P(x)$	+	0	-	0	+	0	-

$S =]-\infty; -4] \cup [1; 2]$

d) $(x+7)(2x+1)(3-x) < 0$

racines $x_1 = -7$ $x_2 = -\frac{1}{2}$ $x_3 = 3$

		-7		$-\frac{1}{2}$		3	
$x+7$	-	0	+		+		+
$2x+1$	-		-	0	+		+
$3-x$	+		+		+	0	-
$P(x)$	+	0	-	0	+	0	-

$S =]-7; -\frac{1}{2}[\cup]3; +\infty[$

$$c) \quad x^3 - 4x \geq 0 \Leftrightarrow x(x^2 - 4) \geq 0 \Leftrightarrow x(x-2)(x+2) \geq 0$$

$$\text{racines: } x_1 = 0 \quad x_2 = 2 \quad x_3 = -2$$

		-2		0		2	
X	-		-	0	+		+
X-2	-		-		-	0	+
X+2	-	0	+		+		+
P(x)	-	0	+	0	-	0	+

$$S = [-2; 0] \cup [2; +\infty[$$

$$f) \quad (2x-5)(3x+4)(1-2x) \geq 0$$

$$\text{racines: } x_1 = \frac{5}{2} \quad x_2 = -\frac{4}{3} \quad x_3 = \frac{1}{2}$$

		$-\frac{4}{3}$		$\frac{1}{2}$		$\frac{5}{2}$	
2x-5	-		-		-	0	+
3x+4	-	0	+		+		+
1-2x	+		+	0	-		-
P(x)	+	0	-	0	+	0	-

$$S =]-\infty; -\frac{4}{3}] \cup [\frac{1}{2}; \frac{5}{2}]$$

Exercise 33

$$a) \quad x^2 + 4 \geq 5x \Leftrightarrow x^2 - 5x + 4 \geq 0 \Leftrightarrow (x-4)(x-1) \geq 0$$

$$\text{racine } x_1 = 4 \quad x_2 = 1$$

		1		4	
X-4	-		-	0	+
X-1	-	0	+		+
P(x)	+	0	-	0	+

$$S =]-\infty; 1] \cup [4; +\infty[$$

$$b) (2x+1)(5-3x) > 0$$

$$\text{racines } x_1 = -\frac{1}{2} \quad x_2 = \frac{5}{3}$$

		$-\frac{1}{2}$		$\frac{5}{3}$	
$2x+1$	-	0	+		+
$5-3x$	+		+	0	-
$P(x)$	-	0	+	0	-

$$S =]-\frac{1}{2}; \frac{5}{3}[$$

$$c) 3x^2 - 7x \leq 2x^2 - 12 \Leftrightarrow x^2 - 7x + 12 \leq 0 \Leftrightarrow$$

$$(x-3)(x-4) \leq 0$$

$$\text{racines } x_1 = 3 \quad x_2 = 4$$

		3		4	
$x-3$	-	0	+		+
$x-4$	-		-	0	+
$P(x)$	+	0	-	0	+

$$S = [3; 4]$$

$$d) (x-2)(5x+7) > (2x-3)(x-2)$$

$$(x-2)(5x+7) - (2x-3)(x-2) > 0$$

$$(x-2)[(5x+7) - (2x-3)] > 0$$

$$(x-2)[5x+7-2x+3] > 0$$

$$(x-2)(3x+10) > 0$$

$$\text{racines: } x_1 = 2 \quad x_2 = -\frac{10}{3}$$

		$-\frac{10}{3}$		2	
$x-2$	-		-	0	+
$3x+10$	-	0	+		+
$P(x)$	+	0	-	0	+

$$S =]-\infty; -\frac{10}{3}[\cup]2; +\infty[$$

$$c) \quad x^3 - 9x < 0 \Leftrightarrow x(x^2 - 9) < 0 \Leftrightarrow x(x-3)(x+3) < 0$$

$$\text{racines: } x_1 = 0 \quad x_2 = 3 \quad x_3 = -3$$

		-3		0		3	
x	-		-	0	+		+
x-3	-		-		-	0	+
x+3	-	0	+		+		+
p(x)	-	0	+	0	-	0	+

$$S =]-\infty; -3[\cup]0; 3[$$

$$f) \quad (2x+1)(x-2) - 4x^2 + 1 \leq 0$$

$$(2x+1)(x-2) - (4x^2 - 1) \leq 0$$

$$(2x+1)(x-2) - (2x+1)(2x-1) \leq 0$$

$$(2x+1) [(x-2) - (2x-1)] \leq 0$$

$$(2x+1) [x-2-2x+1] \leq 0$$

$$(2x+1)(-x-1) \leq 0$$

$$\text{racines } x_1 = -\frac{1}{2} \quad x_2 = -1$$

		-1		-\frac{1}{2}	
2x+1	-		-	0	+
-x-1	+	0	-		-
p(x)	-	0	+	0	-

$$S =]-\infty; -1] \cup [-\frac{1}{2}; +\infty[$$

Exercice 34

a) $4x^2 + 7x - 2 \geq 0$

• Calculer les racines : $a=4$ $b=7$ $c=-2$

$$\Delta = 7^2 - 4 \cdot 4 \cdot (-2) = 49 + 32 = 81 \Rightarrow \sqrt{\Delta} = \sqrt{81} = 9$$

$$x_{1,2} = \frac{-7 \pm 9}{2 \cdot 4} = \frac{-7 \pm 9}{8} \begin{cases} x_1 = \frac{2}{8} = \frac{1}{4} \\ x_2 = -\frac{16}{8} = -2 \end{cases}$$

$$P(x) = 4(x - \frac{1}{4})(x + 2)$$

		-2		$\frac{1}{4}$	
4	+		+		+
$x - \frac{1}{4}$	-		-	0	+
$x + 2$	-	0	+		+
$P(x)$	+	0	-	0	+

$$S =]-\infty; -2] \cup [\frac{1}{4}; +\infty[$$

b) $5x^2 + 13x - 6 > 0$

Calculer les racines : $\Delta = 13^2 - 4 \cdot 5 \cdot (-6) = 169 + 120 = 289 \Rightarrow \sqrt{289} = 17$

$$x_{1,2} = \frac{-13 \pm 17}{2 \cdot 5} = \frac{-13 \pm 17}{10} \begin{cases} x_1 = \frac{4}{10} = \frac{2}{5} \\ x_2 = -\frac{30}{10} = -3 \end{cases}$$

$$P(x) = 5(x - \frac{2}{5})(x + 3)$$

		-3		$\frac{2}{5}$	
5	+		+		+
$x - \frac{2}{5}$	-		-	0	+
$x + 3$	-	0	+		+
$P(x)$	+	0	-	0	+

$$S =]-\infty; -3[\cup]\frac{2}{5}; +\infty[$$

c) $x^2 \leq -4x - 2 \Leftrightarrow x^2 + 4x + 2 \leq 0$

Calculer les racines : $\Delta = 4^2 - 4 \cdot 1 \cdot 2 = 16 - 8 = 8 \Rightarrow \sqrt{\Delta} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

$$x_{1,2} = \frac{-4 \pm 2\sqrt{2}}{2} = \cancel{2} \frac{(-2 \pm \sqrt{2})}{\cancel{2}} = -2 \pm \sqrt{2} \begin{cases} x_1 = -2 + \sqrt{2} \\ x_2 = -2 - \sqrt{2} \end{cases}$$

$$P(x) = (x - (-2 + \sqrt{2}))(x - (-2 - \sqrt{2})) = (x + 2 - \sqrt{2})(x + 2 + \sqrt{2})$$

		$-2 - \sqrt{2}$		$-2 + \sqrt{2}$	
$x + 2 - \sqrt{2}$	-		-	0	+
$x + 2 + \sqrt{2}$	-	0	+		+
$P(x)$	+	0	-	0	+

$$S = [-2 - \sqrt{2}; -2 + \sqrt{2}]$$

$$d) \quad x^2 + 6x + 5 > 2x^2 + 8x - 10 \Leftrightarrow -x^2 - 2x + 15 > 0$$

$$\text{Calculer les racines: } \Delta = (-2)^2 - 4 \cdot (-1) \cdot 15 = 4 + 60 = 64 \Rightarrow \sqrt{\Delta} = \sqrt{64} = 8$$

$$x_{1,2} = \frac{2 \pm 8}{-1 \cdot 2} = \frac{2 \pm 8}{-2} \quad \begin{array}{l} \nearrow x_1 = -\frac{10}{2} = -5 \\ \searrow x_2 = -\frac{-6}{-2} = 3 \end{array}$$

$$P(x) = -(x+5)(x-3)$$

		-5		3	
-1	-		-		-
x+5	-	0	+		+
x-3	-		-	0	+
P(x)	-	0	+	0	-

$$S =]-5; 3[$$

$$e) \quad 4x^2 + 81 < 36x \Leftrightarrow 4x^2 - 36x + 81 < 0$$

$$\text{Calculer les racines: } \Delta = (-36)^2 - 4 \cdot 4 \cdot 81 = 0$$

$$x_{1,2} = \frac{36}{2 \cdot 4} = \frac{36}{8} = \frac{9}{2}$$

$$P(x) = 4\left(x - \frac{9}{2}\right)^2$$

		$\frac{9}{2}$	
4	+		+
$x - \frac{9}{2}$	-	0	+
$x - \frac{9}{2}$	-	0	+
P(x)	+	0	+

$$S = \emptyset$$

$$f) \quad 24x + 9 \geq -16x^2 \Leftrightarrow 16x^2 + 24x + 9 \geq 0$$

$$\text{Calculer les racines: } \Delta = 24^2 - 4 \cdot 16 \cdot 9 = 0$$

$$x_{1,2} = \frac{-24}{2 \cdot 16} = \frac{-24}{32} = -\frac{3}{4}$$

$$P(x) = 16\left(x + \frac{3}{4}\right)^2$$

		$-\frac{3}{4}$	
16	+		+
$x + \frac{3}{4}$	-	0	+
$x + \frac{3}{4}$	-		+
P(x)	+	0	+

$$S = \mathbb{R} =]-\infty; +\infty[$$

Exercise 35

$$a) \quad 9x^2 + 6x - 2 \leq 0$$

$$\text{Calculer les racines: } \Delta = 6^2 - 4 \cdot 9 \cdot (-2) = 36 + 72 = 108$$

$$\sqrt{\Delta} = \sqrt{108} = \sqrt{3 \cdot 36} = \sqrt{3} \cdot \sqrt{36} = \sqrt{3} \cdot 6 = 6\sqrt{3}$$

$$x_{1,2} = \frac{-6 \pm 6\sqrt{3}}{2 \cdot 9} = \frac{-6 \pm 6\sqrt{3}}{18} = \frac{-1 \pm \sqrt{3}}{3} \quad \begin{array}{l} \nearrow x_1 = \frac{-1 + \sqrt{3}}{3} \\ \searrow x_2 = \frac{-1 - \sqrt{3}}{3} \end{array}$$

$$P(x) = 9\left(x - \left(\frac{-1 + \sqrt{3}}{3}\right)\right)\left(x - \left(\frac{-1 - \sqrt{3}}{3}\right)\right)$$

$$S = \left[\frac{-1 - \sqrt{3}}{3}, \frac{-1 + \sqrt{3}}{3}\right]$$

		$\frac{-1 - \sqrt{3}}{3}$		$\frac{-1 + \sqrt{3}}{3}$	
9	+		+		+
$x - \left(\frac{-1 + \sqrt{3}}{3}\right)$	-		-	0	+
$x - \left(\frac{-1 - \sqrt{3}}{3}\right)$	-	0	+		+
P(x)	+	0	-	0	+

$$b) \quad x^2 < 3x - 1 \Leftrightarrow x^2 - 3x + 1 < 0$$

Calculer les racines: $\Delta = (-3)^2 - 4 \cdot 1 \cdot 1 = 9 - 4 = 5 \quad \sqrt{\Delta} = \sqrt{5}$

$$x_{1,2} = \frac{3 \pm \sqrt{5}}{2} \quad \begin{array}{l} \nearrow x_1 = \frac{3 + \sqrt{5}}{2} \\ \searrow x_2 = \frac{3 - \sqrt{5}}{2} \end{array}$$

$$P(x) = \left(x - \frac{3 + \sqrt{5}}{2}\right) \left(x - \frac{3 - \sqrt{5}}{2}\right)$$

		$\frac{3 - \sqrt{5}}{2}$		$\frac{3 + \sqrt{5}}{2}$	
$x - \frac{3 + \sqrt{5}}{2}$	-		-	0	+
$x - \frac{3 - \sqrt{5}}{2}$	-	0	+		+
$P(x)$	+		-		+

$$S = \left] \frac{3 - \sqrt{5}}{2}; \frac{3 + \sqrt{5}}{2} \right[$$

$$c) \quad 16x^2 - 40x + 25 > 0$$

Calculer les racines: $\Delta = (-40)^2 - 4 \cdot 16 \cdot 25 = 0$

$$x_{1,2} = \frac{40}{2 \cdot 16} = \frac{40}{32} = \frac{5}{4}$$

$$P(x) = 16 \left(x - \frac{5}{4}\right)^2$$

$$S = \mathbb{R} \setminus \left\{ \frac{5}{4} \right\} = \left] -\infty; \frac{5}{4} \right[\cup \left] \frac{5}{4}; +\infty \right[$$

		$\frac{5}{4}$	
16	+		+
$x - \frac{5}{4}$	-	0	+
$x - \frac{5}{4}$	-	0	+
$P(x)$	+	0	+

$$d) \quad -7x^2 + 5x > -2 \Leftrightarrow -7x^2 + 5x + 2 > 0$$

Calculer les racines: $\Delta = 5^2 - 4 \cdot (-7) \cdot 2 = 25 + 56 = 81 \quad \sqrt{81} = 9$

$$x_{1,2} = \frac{-5 \pm 9}{2 \cdot (-7)} = \frac{-5 \pm 9}{-14} = \frac{5 \mp 9}{14} \quad \begin{array}{l} \nearrow x_1 = \frac{-4}{14} = -\frac{2}{7} \\ \searrow x_2 = \frac{14}{14} = 1 \end{array}$$

$$P(x) = -7 \left(x + \frac{2}{7}\right) (x - 1)$$

		$-\frac{2}{7}$		1	
-7	-		-		-
$x + \frac{2}{7}$	-	0	+		+
$x - 1$	-		-	0	+
$P(x)$	-	0	+	0	-

$$S = \left] -\frac{2}{7}; 1 \right[$$

$$e) \quad 3x^2 < 7 \Leftrightarrow 3x^2 - 7 < 0$$

Calculer les racines: $\Delta = -4 \cdot 3 \cdot (-7) = 84 \quad \sqrt{84} = \sqrt{4 \cdot 21} = \sqrt{4} \cdot \sqrt{21} = 2\sqrt{21}$

$$x_{1,2} = \pm \frac{2\sqrt{21}}{2 \cdot 3} = \pm \frac{\sqrt{21}}{3} \quad \begin{array}{l} \nearrow x_1 = \frac{\sqrt{21}}{3} \\ \searrow x_2 = -\frac{\sqrt{21}}{3} \end{array}$$

$$P(x) = 3 \left(x - \frac{\sqrt{21}}{3}\right) \left(x + \frac{\sqrt{21}}{3}\right)$$

		$-\frac{\sqrt{21}}{3}$		$\frac{\sqrt{21}}{3}$	
3	+		+		+
$x - \frac{\sqrt{21}}{3}$	-		-	0	+
$x + \frac{\sqrt{21}}{3}$	-	0	+		+
$P(x)$	+	0	-	0	+

$$S = \left] -\frac{\sqrt{21}}{3}; \frac{\sqrt{21}}{3} \right[$$

$$f) 2x - 1 \geq -2x^2 \Leftrightarrow 2x^2 + 2x - 1 \geq 0$$

Calculer les racines: $\Delta = 2^2 - 4 \cdot 2 \cdot (-1) = 4 + 8 = 12$

$$\sqrt{\Delta} = \sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

$$x_{1,2} = \frac{-2 \pm 2\sqrt{3}}{2 \cdot 2} = \frac{\cancel{2}(-1 \pm \sqrt{3})}{\cancel{2} \cdot 2} = \frac{-1 \pm \sqrt{3}}{2} \begin{matrix} \nearrow x_1 = \frac{-1 + \sqrt{3}}{2} \\ \searrow x_2 = \frac{-1 - \sqrt{3}}{2} \end{matrix} \quad P(x) = 2\left(x - \frac{-1 + \sqrt{3}}{2}\right)\left(x - \frac{-1 - \sqrt{3}}{2}\right)$$

		$\frac{-1 - \sqrt{3}}{2}$		$\frac{-1 + \sqrt{3}}{2}$	
2	+		+		+
$x - \frac{-1 + \sqrt{3}}{2}$	-		-	0	+
$x - \frac{-1 - \sqrt{3}}{2}$	-	0	+		+
P(x)	+	0	-	0	+

$$S =]-\infty; \frac{-1 - \sqrt{3}}{2}] \cup [\frac{-1 + \sqrt{3}}{2}; +\infty[$$

Exercice 36

Soit x la longueur du segment AM [cm] $0 \leq x \leq 4$

$4 - x$: longueur du segment MB [cm]

$$x^2 + (4 - x)^2 \geq 10$$

$$x^2 + 16 - 8x + x^2 \geq 10$$

$$2x^2 - 8x + 6 \geq 0$$

$$2(x^2 - 4x + 3) \geq 0 \Leftrightarrow 2(x - 3)(x - 1) \geq 0 \quad \text{racines: } 3 \text{ et } 1$$

	0	1		3	4
$x - 3$	-		-	0	+
$x - 1$	-	0	+		+
P(x)	+	0	-	0	+

$$S = [0; 1] \cup [3; 4]$$

Comme $0 \leq x \leq 4$, AM mesure entre 0 et 1 cm ou entre 3 et 4 cm.

Exercice 37

Soit x la longueur du segment AG [cm] $0 \leq x \leq 10$

$$FI = 10 - x$$

$$x^2 + (10 - x)^2 < 58 \quad | \quad CL$$

$$x^2 + 100 - 20x + x^2 < 58 \quad | \quad CL$$

$$2x^2 - 20x + 100 < 58 \quad | \quad -58$$

$$2x^2 - 20x + 42 < 0$$

$$2(x^2 - 10x + 21) < 0 \quad | \quad \text{fact.}$$

$$2(x - 3)(x - 7) < 0$$

$$\swarrow \quad \searrow$$
$$x = 3 \quad x = 7$$

	$-\infty$	3		7	$+\infty$
2	+	+	+	+	+
$x - 3$	-	0	+	+	+
$x - 7$	-	-	-	0	+
$2(x - 3)(x - 7)$	+	0	-	0	+

$$S =]3; 7[$$

AG doit être plus grand que 3cm, mais plus petit que 7cm.