

Corrigés: Factorisation - Applications

Exercice 17

$$a) \frac{12x - 13y}{24x^2 - 26xy} = \frac{\cancel{12x} - \cancel{13y}}{2x(\cancel{12x} - \cancel{13y})} = \frac{1}{2x}$$

$$b) \frac{a^2 - ab}{4a - 4ab} = \frac{\cancel{a}(a-b)}{4\cancel{a}(1-b)} = \frac{a-b}{4-4b}$$

$$c) \frac{4x + 10y}{5y + 2x} = \frac{2(\cancel{2x} + \cancel{5y})}{\cancel{2x} + \cancel{5y}} = 2$$

$$d) \frac{x-y}{3y-x} \quad \text{on ne peut pas simplifier!}$$

$$e) \frac{1-x}{1-2x+x^2} = \frac{\cancel{1-x}}{(1-x)^2} = \frac{1}{1-x}$$

$$f) \frac{16x^2 - 25y^2}{8x - 10y} = \frac{(4x+5y)(\cancel{4x-5y})}{2(\cancel{4x-5y})} = \frac{4x+5y}{2}$$

$$g) \frac{x^2 - y^2}{x-y} = \frac{(x+y)(\cancel{x-y})}{\cancel{x-y}} = x+y$$

$$h) \frac{x^2 + 2xy + y^2}{ax + ay} = \frac{(x+y)^2}{a(\cancel{x+y})} = \frac{x+y}{a}$$

$$i) \frac{x^2 - 1}{2x + 2} = \frac{(\cancel{x+1})(x-1)}{2(\cancel{x+1})} = \frac{x-1}{2}$$

$$j) \frac{36x^3 - x}{6x^2 - x} = \frac{\cancel{x}(36x^2 - 1)}{\cancel{x}(6x - 1)} = \frac{(6x+1)(\cancel{6x-1})}{\cancel{6x-1}} = 6x+1$$

$$k) \frac{c^2 - ac}{a^2 - ac} = \frac{c(c-a)}{a(a-c)} = \frac{-c(\cancel{a-c})}{a(\cancel{a-c})} = -\frac{c}{a}$$

$$l) \frac{a^2 - 2ab + b^2}{a^2 - b^2} = \frac{(a-b)^2}{(a+b)(\cancel{a-b})} = \frac{a-b}{a+b}$$

Exercice 18

$$a) \frac{24a^2b^2c + 12b^2}{12ab^3c^2 - 6b^2} = \frac{\cancel{12b^2}(2a^2c + 1)}{\cancel{6b^2}(2abc^2 - 1)} = \frac{2(2a^2c + 1)}{2abc^2 - 1}$$

$$b) \frac{1-x^2}{1+x} = \frac{(\cancel{1+x})(1-x)}{\cancel{1+x}} = 1-x$$

$$c) \frac{9x^2 + 12x + 4}{3x + 2} = \frac{(3x+2)^2}{\cancel{3x+2}} = 3x+2$$

$$d) \frac{a^2 - 4}{5ab + 10b} = \frac{(a+2)(a-2)}{5b(a+2)} = \frac{a-2}{5b}$$

$$e) \frac{25x^2 - 9y^2}{25x^2 - 30xy + 9y^2} = \frac{(5x+3y)(5x-3y)}{(5x-3y)^2} = \frac{5x+3y}{5x-3y}$$

$$f) \frac{x^3 - 9y^2x}{2x^2 - 12xy + 18y^2} = \frac{x(x^2 - 9y^2)}{2(x^2 - 6xy + 9y^2)} = \frac{x(x+3y)(x-3y)}{2(x-3y)^2} = \frac{x(x+3y)}{2(x-3y)}$$

Exercise 19

$$a) \frac{3a}{2b} : \frac{2a}{2b} = \frac{3a}{2b} \cdot \frac{1}{2a} = \frac{3}{4b}$$

$$b) \frac{(a+b)^2}{c} \cdot \frac{c}{a^2 - b^2} = \frac{(a+b)^2 \cdot c}{a^2 - b^2} = \frac{(a+b)^2 \cdot c}{(a+b)(a-b)} = \frac{c \cdot (a+b)}{a-b}$$

$$c) \frac{a+b}{a+1} : \frac{ca+cb}{2a+2} = \frac{a+b}{a+1} \cdot \frac{2a+2}{ca+cb} = \frac{(a+b) \cdot 2(a+1)}{(a+1) \cdot c \cdot (a+b)} = \frac{2}{c}$$

$$d) \frac{4(a-4)}{5} : \frac{2a-8}{5} = \frac{4(a-4)}{5} \cdot \frac{5}{2a-8} = \frac{4 \cdot (a-4) \cdot 5}{8(a-4)} = 10$$

$$e) \frac{p^2 - q^2}{p^2 + 2pq + q^2} \cdot \frac{p+q}{p-q} = \frac{(p+q)(p-q)}{(p+q)^2} \cdot \frac{(p+q)}{(p-q)} = 1$$

$$f) \frac{w^2 - w - 12}{t^2} : \frac{w-4}{t^2 - t} = \frac{w^2 - w - 12}{t^2} \cdot \frac{t^2 - t}{w-4} = \frac{(w+3)(w-4)}{t^2} \cdot \frac{t(t-1)}{t \cdot (w-4)} = \frac{(w+3)(t-1)}{t}$$

Exercise 20

$$a) \frac{x^2 - 3x - 10}{x^2 - 6x + 5} = \frac{(x-5)(x+2)}{(x-5)(x-1)} = \frac{x+2}{x-1}$$

$$b) \frac{x^2 - 4x - 21}{x^2 + 10x + 21} = \frac{(x-7)(x+3)}{(x+7)(x+3)} = \frac{x-7}{x+7}$$

$$c) \frac{x^2 - 1}{x^2 + 3x - 4} = \frac{(x+1)(x-1)}{(x+4)(x-1)} = \frac{x+1}{x+4}$$

$$d) \frac{x^2 + 6x - 16}{x^2 + 3x - 10} = \frac{(x+8)(x-2)}{(x+5)(x-2)} = \frac{x+8}{x+5}$$

$$e) \frac{2x^2 - 2x - 12}{3x^2 - 12x - 36} = \frac{2(x^2 - x - 6)}{3(x^2 - 4x - 12)} = \frac{2(x-3)(x+2)}{3(x-6)(x+2)} = \frac{2(x-3)}{3(x-6)}$$

$$f) \frac{5x^2 + 15x - 20}{5x^2 + 5x - 60} = \frac{5(x^2 + 3x - 4)}{5(x^2 + x - 12)} = \frac{(x+4)(x-1)}{(x+4)(x-3)} = \frac{x-1}{x-3}$$

$$g) \frac{5x^2 - 5x}{3x^2 - 3} = \frac{5x(x-1)}{3(x^2 - 1)} = \frac{5x(x-1)}{3(x+1)(x-1)} = \frac{5x}{3(x+1)}$$

$$h) \frac{2x^3 - 8x^2 + 8x}{8x^2(x-2)^3} = \frac{2x(x^2 - 4x + 4)}{4 \cdot 8x^2(x-2)^3} = \frac{(x-2)^2}{4x(x-2)^3} = \frac{1}{4x(x-2)}$$

Exercise 22

$$a) \quad 6x^2 + 12x = 0 \Leftrightarrow 6x(x+2) = 0 \Leftrightarrow \begin{array}{l} 6x = 0 \\ x = 0 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x+2 = 0 \\ x = -2 \end{array} \quad \left| \begin{array}{l} -2 \\ \end{array} \right.$$

$$S = \{-2; 0\}$$

$$b) \quad x^2 - 9 = 0 \Leftrightarrow (x+3)(x-3) = 0 \Leftrightarrow \begin{array}{l} x+3 = 0 \\ x = -3 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x-3 = 0 \\ x = 3 \end{array}$$

$$S = \{-3; 3\}$$

$$c) \quad x^5 - 5x^4 = 0 \Leftrightarrow x^4(x-5) = 0 \Leftrightarrow \begin{array}{l} x^4 = 0 \\ x = 0 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x-5 = 0 \\ x = 5 \end{array}$$

$$S = \{0; 5\}$$

$$d) \quad x^2 + 4x - 12 = 0 \Leftrightarrow (x+6)(x-2) = 0 \Leftrightarrow \begin{array}{l} x+6 = 0 \\ x = -6 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x-2 = 0 \\ x = 2 \end{array}$$

$$S = \{-6; 2\}$$

$$e) \quad x^2 + 2x = 35 \Leftrightarrow x^2 + 2x - 35 = 0 \Leftrightarrow (x+7)(x-5) = 0 \Leftrightarrow \begin{array}{l} x+7 = 0 \\ x = -7 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x-5 = 0 \\ x = 5 \end{array}$$

$$S = \{-7; 5\}$$

$$f) \quad x^2 + 5x = 24 \Leftrightarrow x^2 + 5x - 24 = 0 \Leftrightarrow (x+8)(x-3) = 0 \Leftrightarrow \begin{array}{l} x+8 = 0 \\ x = -8 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x-3 = 0 \\ x = 3 \end{array}$$

$$S = \{-8; 3\}$$

$$g) \quad 5x^2 + 15x + 10 = 0 \Leftrightarrow 5(x^2 + 3x + 2) = 0 \Leftrightarrow 5(x+2)(x+1) = 0$$

$$\Leftrightarrow \begin{array}{l} x+2 = 0 \\ x = -2 \end{array} \quad \underline{\text{ou}} \quad \begin{array}{l} x+1 = 0 \\ x = -1 \end{array}$$

$$S = \{-2; -1\}$$

$$h) \quad 4x^2 - 4x + 1 = 0 \Leftrightarrow (2x-1)^2 = 0 \Leftrightarrow \begin{array}{l} 2x-1 = 0 \\ 2x = 1 \\ x = \frac{1}{2} \end{array} \quad \left| \begin{array}{l} +1 \\ :2 \end{array} \right.$$

$$S = \left\{ \frac{1}{2} \right\}$$

Exercice 23

a) Dénominateur commun entre x et x^2 est x^2 .

$$\frac{3}{x} + \frac{4}{x^2} = \frac{3}{x} \cdot \frac{x}{x} + \frac{4}{x^2} = \frac{3x}{x^2} + \frac{4}{x^2} = \frac{3x+4}{x^2}$$

b) Dénominateur commun entre y et x est xy .

$$\frac{2x}{y} + \frac{y}{x} = \frac{2x}{y} \cdot \frac{x}{x} + \frac{y}{x} \cdot \frac{y}{y} = \frac{2x^2}{xy} + \frac{y^2}{xy} = \frac{2x^2+y^2}{xy}$$

c) Dénominateur commun entre $3x$ et x^2 est $3x^2$.

$$\frac{5}{3x} - \frac{2}{x^2} = \frac{5}{3x} \cdot \frac{x}{x} - \frac{2}{x^2} \cdot \frac{3}{3} = \frac{5x}{3x^2} - \frac{6}{3x^2} = \frac{5x-6}{3x^2}$$

d) Dénom. commun entre x et $x^2+x = x(x+1)$ est $x(x+1)$

$$\frac{1}{x} - \frac{1}{x(x+1)} = \frac{1}{x} \cdot \frac{x+1}{x+1} - \frac{1}{x(x+1)} = \frac{x+1}{x(x+1)} - \frac{1}{x(x+1)} = \frac{x+1-1}{x(x+1)} = \frac{x}{x(x+1)} = \frac{1}{x+1}$$

$$e) \frac{4}{z-1} + \frac{z}{z^2-1} = \frac{4}{z-1} + \frac{z}{(z+1)(z-1)} = \frac{4(z+1)}{(z+1)(z-1)} + \frac{z}{(z+1)(z-1)} = \frac{4z+4+z}{(z+1)(z-1)} = \frac{5z+4}{(z+1)(z-1)}$$

$$f) \frac{a}{a^2-4} - \frac{a}{a^2-4a+4} = \frac{a}{(a+2)(a-2)} - \frac{a}{(a-2)^2} = \frac{a(a-2) - a(a+2)}{(a+2)(a-2)^2} = \frac{a^2-2a-a^2-2a}{(a+2)(a-2)^2} = \frac{-4a}{(a+2)(a-2)^2}$$

$$g) \frac{2}{2w+1} - \frac{2}{4w^2-1} + \frac{1}{2w-1} = \frac{2}{2w+1} - \frac{2}{(2w+1)(2w-1)} + \frac{1}{2w-1} = \frac{2(2w-1) - 2 + 1 \cdot (2w+1)}{(2w+1)(2w-1)} = \frac{4w-2-2+2w+1}{(2w+1)(2w-1)} = \frac{6w-3}{(2w+1)(2w-1)} = \frac{3(2w-1)}{(2w+1)(2w-1)} = \frac{3}{2w+1}$$

$$h) \frac{1}{x} - \frac{1}{x^2+x} - \frac{1}{x^2+2x+1} = \frac{1}{x} - \frac{1}{x(x+1)} - \frac{1}{(x+1)^2} = \frac{(x+1)^2 - (x+1) - x}{x(x+1)^2} = \frac{x^2+2x+1-x-1-x}{x(x+1)^2} = \frac{x^2-1-x}{x(x+1)^2} = \frac{x}{(x+1)^2}$$

$$i) \frac{3}{x+y} + \frac{2}{x} = \frac{3x+2(x+y)}{x(x+y)} = \frac{3x+2x+2y}{x(x+y)} = \frac{5x+2y}{x(x+y)}$$

$$j) \frac{b-a}{6a+2b} - \frac{a+b}{3a+b} = \frac{b-a}{2(3a+b)} - \frac{a+b}{3a+b} = \frac{b-a-2(a+b)}{2(3a+b)} = \frac{b-a-2a-2b}{2(3a+b)} = \frac{-3a-b}{2(3a+b)} = -\frac{1}{2}$$

Exercise 24

$$a) \frac{5x}{2y} - \frac{3y}{10x} = \frac{5x \cdot 10x - 3y \cdot 2y}{2y \cdot 10x} = \frac{50x^2 - 6y^2}{20xy} = \frac{2(25x^2 - 3y^2)}{20xy} = \frac{25x^2 - 3y^2}{10xy}$$

$$b) \frac{ab - b^2}{a^3 + a^2} : \frac{a^2b - b^3}{a^2 + a} = \frac{b(a-b)}{a^2(a+1)} \cdot \frac{a^2 + a}{a^2b - b^3} = \frac{b(a-b)}{a^2(a+1)} \cdot \frac{a(a+1)}{b(a^2 - b^2)} = \frac{a-b}{a(a^2 - b^2)} = \frac{a-b}{a(a+b)(a-b)} = \frac{1}{a(a+b)}$$

$$c) \frac{a}{2b} - \frac{b}{5c} = \frac{5ac - 2b^2}{2b \cdot 5c} = \frac{5ac - 2b^2}{10bc}$$

$$d) \frac{18m^2np}{18m^2np + 54m^2np^2} = \frac{18m^2np}{18m^2np(n+3p)} = \frac{1}{n+3p}$$

$$e) (2a-b) \cdot \frac{1}{b-2a} = -(b-2a) \cdot \frac{1}{b-2a} = -1$$

$$f) \frac{a^2 + 6a + 9}{4} : (2a+6) = \frac{(a+3)^2}{4} \cdot \frac{1}{2a+6} = \frac{(a+3)^2}{4 \cdot 2(a+3)} = \frac{a+3}{8}$$

$$g) \frac{5}{x-3} - \frac{7}{x+5} + \frac{3x-40}{x^2+2x-15} = \frac{5}{x-3} - \frac{7}{x+5} + \frac{3x-40}{(x+5)(x-3)} = \frac{5(x+5) - 7(x-3) + 3x-40}{(x+5)(x-3)} = \frac{5x+25-7x+21+3x-40}{(x+5)(x-3)} = \frac{x+6}{(x+5)(x-3)}$$

$$h) \frac{3(x+4)}{8xy} \cdot \frac{x^2-16}{x^2y^2} = \frac{3(x+4)}{8xy} \cdot \frac{x^2y^2}{x^2-16} = \frac{3(x+4) \cdot x \cdot y}{8 \cdot (x+4)(x-4)} = \frac{3xy}{8(x-4)}$$

Exercise 25

$$a) \frac{\frac{a}{a^2-1}}{\frac{1}{a+1} - \frac{1}{a-1}} = \frac{\frac{a}{(a+1)(a-1)}}{\frac{(a-1) - (a+1)}{(a+1)(a-1)}} = \frac{\frac{a}{(a+1)(a-1)}}{\frac{a-1-a-1}{(a+1)(a-1)}} = \frac{\frac{a}{(a+1)(a-1)}}{\frac{-2}{(a+1)(a-1)}} = \frac{a}{-2} = -\frac{a}{2}$$

$$b) \frac{\frac{m-n}{m+n} - \frac{m}{m-n}}{\frac{m+n}{m-n} - \frac{n}{m+n}} = \frac{\frac{(m-n)^2 - m(m+n)}{(m+n)(m-n)}}{\frac{(m+n)^2 - n(m-n)}{(m+n)(m-n)}} = \frac{(m-n)^2 - m(m+n)}{(m+n)^2 - n(m-n)} \cdot \frac{(m+n)(m-n)}{(m+n)^2 - n(m-n)} = \frac{m^2 - 2mn + n^2 - m^2 - mn}{m^2 + 2mn + n^2 - mn + n^2} = \frac{n^2 - 3mn}{m^2 + mn + 2n^2}$$

$$c) \frac{\frac{a}{a+1} - \frac{a}{a+2}}{\frac{a}{a+2} - \frac{2a}{a-1}} = \frac{\frac{a(a+2) - a(a+1)}{(a+1)(a+2)}}{\frac{a(a-1) - 2a(a+2)}{(a+2)(a-1)}} = \frac{\frac{a^2+2a-a^2-a}{(a+1)(a+2)}}{\frac{a^2-a-2a^2-4a}{(a+2)(a-1)}} = \frac{\frac{a}{(a+1)(a+2)}}{\frac{-a^2-5a}{(a+2)(a-1)}} = \frac{a}{(a+1)(a+2)} \cdot \frac{(a+2)(a-1)}{-a^2-5a} = \frac{a(a-1)}{(a+1)(a+2)(-a-5)} = \frac{-(a-1)}{(a+1)(a+5)} = \frac{1-a}{(a+1)(a+5)}$$