

Corrigés: Équations quadratiques

Exercice 26

a) $2x^2 = 50 \Leftrightarrow 2x^2 - 50 = 0$

$a=2 \quad b=0 \quad c=-50$

$\Delta = b^2 - 4ac = 0^2 - 4 \cdot 2 \cdot (-50) = 400$

$\Rightarrow \sqrt{\Delta} = \sqrt{400} = 20$

$x_{1,2} = \frac{0 \pm 20}{2 \cdot 2} = \pm \frac{20}{4} \rightarrow \begin{matrix} x_1 = 5 \\ x_2 = -5 \end{matrix}$

$S = \{-5; 5\}$

$P(x) = 2 \cdot (x-5)(x+5)$

b) $4x^2 - 9x = 0 \Leftrightarrow x(4x-9) = 0$

$\swarrow \quad \searrow$
 $x=0 \quad \text{ou} \quad 4x-9=0$

$x = \frac{9}{4}$

$S = \{0; \frac{9}{4}\}$

$P(x) = x(4x-9) = 4x(x - \frac{9}{4})$

c) $x^2 + 6x = -9 \Leftrightarrow x^2 + 6x + 9 = 0$

$a=1 \quad b=6 \quad c=9$

$\Delta = 6^2 - 4 \cdot 1 \cdot 9 = 36 - 36 = 0$

$x_1 = \frac{-6}{2} = -3$

$S = \{-3\}$

$P(x) = (x+3)^2$

d) $-\frac{3}{4}x^2 + \frac{5}{2}x = 4 \Leftrightarrow -\frac{3}{4}x^2 + \frac{5}{2}x - 4 = 0 \quad | \cdot 4$

$\Leftrightarrow -3x^2 + 10x - 16 = 0$

$a=-3 \quad b=10 \quad c=-16$

$\Delta = 10^2 - 4 \cdot (-3) \cdot (-16) = -92$

$\Delta < 0 \Rightarrow$ pas de solution $S = \emptyset$

$P(x)$ ne peut pas être factorisé.

$$e) 3x^2 - 48 = 0$$

$$a=3 \quad b=0 \quad c=-48$$

$$\Delta = 0^2 - 4 \cdot 3 \cdot (-48) = 576$$

$$\sqrt{\Delta} = \sqrt{576} = 24$$

$$x_{1,2} = \frac{\pm 24}{2 \cdot 3} = \pm \frac{24}{6} \begin{matrix} \nearrow 4 \\ \searrow -4 \end{matrix}$$

$$S = \{-4, 4\}$$

$$P(x) = 3(x+4)(x-4)$$

$$f) 4x^2 + \frac{1}{4}x = 5x - 3 \quad | \cdot 4 \Leftrightarrow 16x^2 + x = 20x - 12$$

$$\Leftrightarrow 16x^2 - 19x + 12 = 0$$

$$a=16 \quad b=-19 \quad c=12$$

$$\Delta = (-19)^2 - 4 \cdot 16 \cdot 12 = -607$$

$\Delta < 0 \Rightarrow S = \emptyset$ et $P(x)$ ne peut pas être factorisé.

$$g) 6x^2 - x = 2 \Leftrightarrow 6x^2 - x - 2 = 0$$

$$a=6 \quad b=-1 \quad c=-2$$

$$\Delta = (-1)^2 - 4 \cdot 6 \cdot (-2) = 1 + 48 = 49$$

$$\sqrt{\Delta} = \sqrt{49} = 7$$

$$x_{1,2} = \frac{1 \pm 7}{2 \cdot 6} = \frac{1 \pm 7}{12} \begin{matrix} \nearrow \frac{8}{12} = \frac{2}{3} \\ \searrow \frac{-6}{12} = -\frac{1}{2} \end{matrix}$$

$$S = \left\{-\frac{1}{2}, \frac{2}{3}\right\}$$

$$P(x) = 6 \cdot \left(x + \frac{1}{2}\right) \left(x - \frac{2}{3}\right) = 2 \cdot \left(x + \frac{1}{2}\right) \cdot 3 \left(x - \frac{2}{3}\right) = (2x+1)(3x-2)$$

$$h) 4x^2 + 81 = 36x \Leftrightarrow 4x^2 - 36x + 81 = 0$$

$$a=4 \quad b=-36 \quad c=81$$

$$\Delta = (-36)^2 - 4 \cdot 4 \cdot 81 = 0$$

$$x_{1,2} = \frac{36}{2 \cdot 4} = \frac{36}{8} = \frac{9}{2}$$

$$S = \left\{\frac{9}{2}\right\}$$

$$P(x) = 4 \left(x - \frac{9}{2}\right)^2 = 2 \left(x - \frac{9}{2}\right) \cdot 2 \left(x - \frac{9}{2}\right) = (2x-9)(2x-9) = (2x-9)^2$$

$$i) 6x^2 - 13x - 5 = 0$$

$$\Delta = (-13)^2 - 4 \cdot 6 \cdot (-5) = 169 + 120 = 289 \quad \sqrt{\Delta} = 17$$

$$x_{1,2} = \frac{13 \pm 17}{2 \cdot 6} = \frac{13 \pm 17}{12} \begin{matrix} \nearrow \frac{-4}{12} = -\frac{1}{3} \\ \searrow \frac{30}{12} = \frac{5}{2} \end{matrix}$$

$$S = \left\{-\frac{1}{3}, \frac{5}{2}\right\}$$

$$P(x) = 6 \left(x + \frac{1}{3}\right) \left(x - \frac{5}{2}\right) = 3 \left(x + \frac{1}{3}\right) \cdot 2 \left(x - \frac{5}{2}\right) = (3x+1)(2x-5)$$

Exercise 27

a) $5x^2 + 13x - 6 = 0$ $\Delta = 13^2 - 4 \cdot 5 \cdot (-6) = 169 + 120 = 289 \Rightarrow \sqrt{\Delta} = 17$
 $x_{1,2} = \frac{-13 \pm 17}{2 \cdot 5} = \frac{-13 + 17}{10} \begin{cases} \frac{4}{10} = \frac{2}{5} \\ -\frac{30}{10} = -3 \end{cases} \quad S = \left\{ -3; \frac{2}{5} \right\}$

b) $14x^2 + 28x + 21 = 0 \Leftrightarrow 7(2x^2 + 4x + 3) = 0$
 $a=2 \quad b=4 \quad c=3 \quad \Delta = 4^2 - 4 \cdot 2 \cdot 3 = 16 - 24 < 0 \quad S = \emptyset$

c) $6x^2 + 10x + 2 = 0 \Leftrightarrow 2(3x^2 + 5x + 1) = 0$
 $\Delta = 5^2 - 4 \cdot 3 \cdot 1 = 25 - 12 = 13$
 $x_{1,2} = \frac{-5 \pm \sqrt{13}}{2 \cdot 3} = \frac{-5 \pm \sqrt{13}}{6} \begin{cases} \frac{-5 - \sqrt{13}}{6} \\ \frac{-5 + \sqrt{13}}{6} \end{cases}$
 $S = \left\{ \frac{-5 - \sqrt{13}}{6}; \frac{-5 + \sqrt{13}}{6} \right\}$

d) $\frac{3}{2}x^2 - 4x - 1 = 0 \quad | \cdot 2 \Leftrightarrow 3x^2 - 8x - 2 = 0$
 $\Delta = (-8)^2 - 4 \cdot 3 \cdot (-2) = 64 + 24 = 88$
 $\sqrt{\Delta} = \sqrt{4 \cdot 22} = \sqrt{4} \cdot \sqrt{22} = 2\sqrt{22}$
 $x_{1,2} = \frac{8 \pm 2\sqrt{22}}{2 \cdot 3} = \frac{4 \pm \sqrt{22}}{3} \begin{cases} \frac{4 - \sqrt{22}}{3} \\ \frac{4 + \sqrt{22}}{3} \end{cases}$
 $S = \left\{ \frac{4 - \sqrt{22}}{3}; \frac{4 + \sqrt{22}}{3} \right\}$

e) $\frac{5}{3}x^2 + 3x + 1 = 0 \quad | \cdot 3 \Leftrightarrow 5x^2 + 9x + 3 = 0$
 $\Delta = 9^2 - 4 \cdot 5 \cdot 3 = 81 - 60 = 21$
 $x_{1,2} = \frac{-9 \pm \sqrt{21}}{2 \cdot 5} = \frac{-9 \pm \sqrt{21}}{10} \begin{cases} \frac{-9 - \sqrt{21}}{10} \\ \frac{-9 + \sqrt{21}}{10} \end{cases}$
 $S = \left\{ \frac{-9 - \sqrt{21}}{10}; \frac{-9 + \sqrt{21}}{10} \right\}$

f) $225x^2 + 210x + 49 = 0$
 $\Delta = 210^2 - 4 \cdot 225 \cdot 49 = 0$
 $x_{1,2} = \frac{-210}{2 \cdot 225} = -\frac{210}{450} = -\frac{21 \cdot 10}{45 \cdot 10} = -\frac{21}{45} = -\frac{7}{15} \quad S = \left\{ -\frac{7}{15} \right\}$

$$g) \quad 6x^2 - 8x - 10 = 0 \quad \Leftrightarrow \quad 2(3x^2 - 4x - 5) = 0$$

$$\Delta = (-4)^2 - 4 \cdot 3 \cdot (-5) = 16 + 60 = 76$$

$$\sqrt{76} = \sqrt{2 \cdot 2 \cdot 19} = \sqrt{2 \cdot 2} \cdot \sqrt{19} = 2\sqrt{19}$$

$$x_{1,2} = \frac{4 \pm 2\sqrt{19}}{2 \cdot 3} = \frac{2 \pm \sqrt{19}}{3} \quad \begin{matrix} \nearrow \frac{2 - \sqrt{19}}{3} \\ \searrow \frac{2 + \sqrt{19}}{3} \end{matrix}$$

$$S = \left\{ \frac{2 - \sqrt{19}}{3}; \frac{2 + \sqrt{19}}{3} \right\}$$

$$h) \quad \begin{array}{l} \frac{1}{2}x + 2x^2 = 3x^2 - \frac{5}{6} \\ 3x + 12x^2 = 18x^2 - 5 \end{array} \quad \begin{array}{l} \cdot 6 \\ -18x^2 + 5 \end{array}$$

$$\Delta = 3^2 - 4 \cdot (-6) \cdot 5 = 9 + 120 = 129$$

$$-6x^2 + 3x + 5 = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{129}}{2 \cdot (-6)} = \frac{3 \mp \sqrt{129}}{12} \quad \begin{matrix} \nearrow \frac{3 - \sqrt{129}}{12} \\ \searrow \frac{3 + \sqrt{129}}{12} \end{matrix}$$

$$S = \left\{ \frac{3 - \sqrt{129}}{12}; \frac{3 + \sqrt{129}}{12} \right\}$$

$$i) \quad 100x^2 + 25x - 200 = 0 \quad \Leftrightarrow \quad 25(4x^2 + x - 8) = 0$$

$$\Delta = 1^2 - 4 \cdot 4 \cdot (-8) = 1 + 128 = 129$$

$$x_{1,2} = \frac{-1 \pm \sqrt{129}}{2 \cdot 4} = \frac{-1 \pm \sqrt{129}}{8} \quad \begin{matrix} \nearrow \frac{-1 - \sqrt{129}}{8} \\ \searrow \frac{-1 + \sqrt{129}}{8} \end{matrix}$$

$$S = \left\{ \frac{-1 - \sqrt{129}}{8}; \frac{-1 + \sqrt{129}}{8} \right\}$$

Exercice 28

Soit x la longueur de l'arête du carré [cm]

$$\begin{array}{rcl} (x-2)(x+5) & = & x^2 + 11 \\ x^2 + 3x - 10 & = & x^2 + 11 \\ 3x - 10 & = & 11 \\ 3x & = & 21 \\ x & = & 7 \end{array} \quad \begin{array}{l} CL \\ -x^2 \\ +10 \\ :3 \end{array}$$

$$x = 7 \quad S = \{7\}$$

La longueur de l'arête du carré mesure 7 cm.

Exercice 29

Soit x le nombre positif cherché

$$x + 48,75 = x^2 \Leftrightarrow 0 = x^2 - x - 48,75$$

$$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-48,75) = 196 \Rightarrow \sqrt{\Delta} = \sqrt{196} = 14$$

$$x_{1,2} = \frac{1 \pm 14}{2} \begin{cases} \frac{15}{2} = 7,5 \\ \frac{-13}{2} = -6,5 \end{cases} \quad S = \{-6,5; 7,5\}$$

à exclure car pas positif.

Le nombre cherché est 7,5.

Exercice 30

Soit r la longueur du rayon d'un cercle [cm]

$$\begin{array}{lcl} (r+30)^2 \cdot \pi & = & 3r^2 \cdot \pi \\ (r+30)^2 & = & 3r^2 \\ r^2 + 60r + 900 & = & 3r^2 \end{array} \quad \begin{array}{l} : \pi \\ \text{CL} \\ -3r^2 \end{array}$$

$$-2r^2 + 60r + 900 = 0$$

$$-2(r^2 - 30r - 450) = 0$$

$$\Delta = (-30)^2 - 4 \cdot 1 \cdot (-450) = 2700$$

$$x_{1,2} = \frac{30 \pm \sqrt{2700}}{2} \begin{cases} -10,981 \\ 40,981 \end{cases}$$

$$S = \{-10,981; 40,981\}$$

à exclure une longueur ne peut pas être négative.

Le rayon mesure 40,981 cm.

Exercice 31

$$a) \begin{cases} x+y=7 \\ xy=12 \end{cases} \Leftrightarrow \begin{cases} x=7-y & (1) \\ (7-y)y=12 & (2) \end{cases}$$

Résoudre (2): $7y - y^2 = 12 \Leftrightarrow 0 = y^2 - 7y + 12$

$$\Delta = (-7)^2 - 4 \cdot 1 \cdot 12 = 49 - 48 = 1 \Rightarrow \sqrt{1} = 1$$

$$y_{1,2} = \frac{7 \pm 1}{2} \begin{cases} y_1 = 4 \\ y_2 = 3 \end{cases}$$

remplacer y_1 et y_2 dans (1)

$$x_1 = 7 - 4 = 3$$

$$x_2 = 7 - 3 = 4$$

$$S = \{(3; 4); (4; 3)\}$$

$$b) \begin{cases} x-y=5 \\ xy=-4 \end{cases} \Leftrightarrow \begin{cases} x=y+5 & (1) \\ (y+5)y=-4 & (2) \end{cases}$$

Résoudre (2): $y^2+5y=-4 \Leftrightarrow y^2+5y+4=0$

$$\Delta = 5^2 - 4 \cdot 1 \cdot 4 = 25 - 16 = 9 \Rightarrow \sqrt{\Delta} = \sqrt{9} = 3$$

$$y_{1,2} = \frac{-5 \pm 3}{2} \begin{cases} y_1 = -4 \\ y_2 = -1 \end{cases} \quad \text{remplacer } y_1 \text{ et } y_2 \text{ dans (1)} \quad \begin{aligned} x_1 &= -4 + 5 = 1 \\ x_2 &= -1 + 5 = 4 \end{aligned}$$

$$S = \{ (1; -4); (4; -1) \}$$

$$c) \begin{cases} x^2+y^2=146 \\ x-y=6 \end{cases} \Leftrightarrow \begin{cases} (y+6)^2+y^2=146 & (1) \\ x=y+6 & (2) \end{cases}$$

Résoudre (1): $y^2+12y+36+y^2=146 \Leftrightarrow 2y^2+12y-110=0$

$$\Leftrightarrow 2(y^2+6y-55)=0 \Leftrightarrow 2(y+11)(y-5)=0$$

$$y+11=0 \quad \text{ou} \quad y-5=0$$

$$y_1 = -11 \quad y_2 = 5$$

remplacer y_1 et y_2 dans (2) $x_1 = -11 + 6 = -5$

$$x_2 = 5 + 6 = 11$$

$$S = \{ (-5; -11); (11; 5) \}$$

$$d) \begin{cases} x^2+y^2=13 \\ x^2-y^2=-5 \end{cases} \Leftrightarrow \begin{cases} (y^2-5)+y^2=13 & (1) \\ x^2=y^2-5 & (2) \end{cases}$$

Résoudre (1): $2y^2-5=13 \Leftrightarrow 2y^2-18=0 \Leftrightarrow 2(y^2-9)=0$

$$\Leftrightarrow 2(y+3)(y-3)=0 \Leftrightarrow y+3=0 \quad \text{ou} \quad y-3=0$$

$$y_1 = -3 \quad y_2 = 3$$

remplacer y_1 dans (2) $x^2 = (-3)^2 - 5 \Leftrightarrow x^2 = 9 - 5 \Leftrightarrow x^2 = 4 \Leftrightarrow$

$$x_1 = 2 \quad \text{ou} \quad x_2 = -2$$

remplacer y_2 dans (2) $x^2 = 3^2 - 5 \Leftrightarrow x^2 = 4 \Leftrightarrow x_3 = 2 \quad \text{ou} \quad x_4 = -2$

$$S = \{ (2; -3); (-2; -3); (2; 3); (-2; 3) \}$$

Challenge

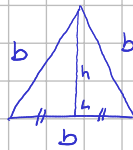
Soit b la largeur du rectangle [cm]

Soit l la longueur du rectangle [cm]

Périmètre: $2l + 3b$

Aire: $b \cdot l + \frac{b \cdot h}{2}$

Soit h la hauteur du triangle équilatéral [cm]



Par Pythagore: $b^2 = \left(\frac{b}{2}\right)^2 + h^2$

$$b^2 = \frac{b^2}{4} + h^2$$

$$\Rightarrow h^2 = b^2 - \frac{b^2}{4} = \frac{4b^2}{4} - \frac{b^2}{4} = \frac{3b^2}{4}$$

$$\Rightarrow h = \sqrt{\frac{3b^2}{4}} = \frac{\sqrt{3}}{\sqrt{4}} \sqrt{b^2} = \frac{\sqrt{3}}{2} b$$

$$\begin{cases} 2l + 3b = 20 \\ bl + \frac{b \cdot \frac{\sqrt{3}}{2} b}{2} = 15,732 \end{cases} \Leftrightarrow \begin{cases} l = \frac{20-3b}{2} & (1) \\ bl + \frac{\sqrt{3} b^2}{4} = 15,732 & (2) \end{cases}$$

Substituer (1) dans (2)

$$b \cdot \left(\frac{20-3b}{2}\right) + \frac{\sqrt{3}}{4} b^2 = 15,732 \Leftrightarrow$$

$$\frac{20b}{2} - \frac{3b^2}{2} + \frac{\sqrt{3}}{4} b^2 = 15,732 \quad | \cdot 4 \Leftrightarrow$$

$$40b - 6b^2 + \sqrt{3} b^2 = 62,928 \Leftrightarrow$$

$$(\sqrt{3}-6)b^2 + 40b - 62,928 = 0$$

$$\Delta = 40^2 - 4 \cdot (\sqrt{3}-6) \cdot (-62,928) = 525,71 \Rightarrow \sqrt{\Delta} = 22,9$$

$$b_{1,2} = \frac{-40 \pm 22,9}{2 \cdot (\sqrt{3}-6)} \quad \begin{cases} b_1 = 2 \\ b_2 \approx 7,37 \end{cases}$$

$$l_1 = \frac{20-3 \cdot 2}{2} = \frac{20-6}{2} = 7$$

$$l_2 = \frac{20-3 \cdot 7,37}{2} = -1,1 < 0 \text{ impossible}$$

$$S = \{(2, 7); (7, 37, -1, 1)\}$$

La longueur mesure 7 cm et la largeur 2 cm.